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UPDATED 7th EDITION

NEW SYLLABUS MATHEMATICS

TEACHER'S RESOURCE BOOK

A Comprehensive Mathematics Programme for Grade 6

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Syllabus Matching Grid
National Curriculum of Pakistan 2022 with New Syllabus Mathematics 1 (Updated 7th Edition)

SLOs	Domain A: Numbers and Operations	Reference
M-06-A01	Find: factors of numbers up to 3 digits multiples of numbers up to 2 digits prime factors of numbers up to 4 digits and express them index notation	Chapter 1
M-06-A02	Identify base and exponent and express numbers given in expanded form in index notation and vice versa	Chapter 1
M-06-A03	Find HCF and LCM of 2 or three numbers (up to 3 digits) using: - prime factorization - division method	Chapter 1
M-06-A04	Solve real-world and problems involving HCF and LCM	Chapter 1
M-06-A05	Recognise, identify, and represent integers (positive, negative, and neutral integers) and their absolute or numerical value	Chapter 2
M-06-A06	Arrange a given list of integers and their absolute values in ascending and descending order	Chapter 2
M-06-A07	Add and subtract up to 2 digits like and unlike integers and verify commutative and associative laws (where applicable)	Chapter 2
M-06-A08	Multiply up to 2 digits like and unlike integers and verify commutative, associative, and distributive laws	Chapter 2
M-06-A09	Divide like and unlike integers	Chapter 2
M-06-A10	Recognise the order of operations and use it to solve mathematical expressions involving whole numbers, decimals, fractions, and integers	Chapter 2
M-06-A11	Express one quantity as a percentage of another, compare two quantities by percentage and increase or decrease a quantity by a given percentage	Chapter 3
M-06-A12	Solve real-world word problems involving percentages	Chapter 3
M-06-A13	Explain rate as a comparison of two quantities where one quantity is 1	Chapter 7
M-06-A14	Calculate ratio of two numbers (up to 3 digits) and simplify ratios	Chapter 7
M-06-A15	Explain and calculate continued ratio	Chapter 7
M-06-A16	Solve real-world problems involving ratio and rate	Chapter 7
M-06-A17	Recognise and calculate squares of numbers up to 2 digits	Chapter 1
M-06-A18	Use language, notation, and Venn Diagrams to represent different types of sets and their elements. (finite, infinite, empty, singleton and universal set)	Chapter 4
SLOs	**Domain B: Algebra**	
M-06-B-01	Recognise simple patterns from various number sequences	Chapter 5
M-06-B-02	Continue a given number sequence and find: - term to term rule - position to term rule	Chapter 5
M-06-B-03	Solve real-life problems involving number sequences and patterns	Chapter 5
M-06-B-04	Explain the term algebra as an extension of arithmetic, where letters, numbers, and symbols are used to construct algebraic expressions	Chapter 6
M-06-B-05	Evaluate algebraic expressions by substitution of variables with numerical values	Chapter 5
M-06-B-06	Manipulate simple algebraic expressions using addition and subtraction	Chapter 5
M-06-B-07	Simplify algebraic expressions	Chapter 5

M-06-B-08	Recognise and construct linear equations in one variable	Chapter 6
M-06-B-09	Solve linear equations involving integers, fractions, and decimal coefficients	Chapter 6
M-06-B-10	Solve realworld problems involving linear equations	Chapter 6
SLOs	**Domain C: Measurement**	
M-06-C-01	Calculate the area of; a path (inside or outside) a rectangle or square, parallelogram, triangle and trapezium	Chapter 8
M-06-C-02	Solve real-life word problems involving perimeter and area	Chapter 8
M-06-C-03	Calculate the surface area and volume of cube and cuboids	Chapter 9
M-06-C-04	Solve real-life word problems involving the surface area and volume of cubes and cuboids	Chapter 9
SLOs	**Domain D: Geometry**	
M-06-D-01	Recognise and identify 3-D shapes (i.e., cube, cuboid, cone, cylinder, sphere, hemisphere, and cone) with respect to their characteristics	Covered in lower level
M-06-D-02	Reflect an object using grid paper and compass and find the line of reflection by construction	Chapter 11
M-06-D-03	Identify and differentiate between parallel lines, perpendicular lines, and transversal	Chapter 10
M-06-D-04	Identify adjacent angles and find unknown angles related to parallel lines and transversals. (corresponding, alternate, and vertically opposite angles)	Chapter 10
M-06-D-05	Recognise rotational symmetry, find the point of rotation and order of rotational symmetry	Chapter 12
M-06-D-06	Construct angles of specific measures (30°, 45°, 60°, 75°, 90°, 105° and 120°) and bisect angles using a compass	Chapter 11
M-06-D-07	Construct a perpendicular (from a point on the line and outside the line) and a perpendicular bisector	Chapter 11
SLOs	**Domain E: Statistics and Probability**	
M-06-E-01	Draw, read, and interpret horizontal and vertical multiple bar graphs and pie charts (including real world problems)	Chapter 13
M-06-E-02	Identify and organise different types of data (i.e., discrete, continuous, grouped and ungrouped)	Chapter 14
M-06-E-03	Calculate the mean, median and mode for ungrouped data and solve related to real-world problems	Chapter 14
M-06-E-04	Explain experiments, outcomes, sample space, events, equally likely events and probability of a single event. Differentiate the outcomes that are equally likely and not equally likely to occur. (including real-world word problems)	Chapter 15

Scheme of Work – New Syllabus Mathematics Book 1 (Updated 7th Edition)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
1	**1** **Primes, Highest Common Factor and Lowest Common Multiple**	**1.1 Prime Numbers (pp. 3 – 9)**	• Explain what a prime number is • Explain how to write a number in index notation • Express a composite number as a product of its prime factors	Identity and use prime numbers	Investigation – Classification of Whole Numbers (p. 4) Thinking Time (p. 4) Investigation – Sieve of Eratosthenes (p. 5) Journal Writing (p. 5) Investigation – Interesting Facts about Prime Numbers (p. 7) Thinking Time (p. 8)			Thinking Time (p. 4) Investigation – Sieve of Eratosthenes (p. 5) Journal Writing (p. 5) Worked Example 3 (p. 7) Practise Now 2 Q 1 – 2 (p. 7) Thinking Time (p. 8)
1		**1.2 Perfect Squares (pp. 9 – 12)**	• Find square of 2-digit numbers using prime factorisation, mental estimation and calculators mental estimation and calculators	Identify and use square numbers Calculate squares,				To determine whether 997 is a prime, it is enough to test whether 997 is divisible by 2, 3, 5, 7, ... or 31 (only 11 prime numbers to test). We do not have to test all the 167 prime numbers. Why?' (p. 11) Ex 1A Q 12 - 131 (p.12)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
2, 3		**1.3 Highest Common Factor and Lowest Common Multiple (pp. 12 – 18)**	• Find the highest common factor (HCF) and lowest common multiple (LCM) of two or more numbers • Solve problems involving HCF and LCM in real-world contexts	Identify and use common factors and common multiples				Practice Now 7 Q 3 (p. 15) Practise Now 11 Q 3 (p. 15) Ex 1B Q6, 9(a) – (e), 10, 11(a) – (d), 13(i), 14(ii) (pp. 17 – 18)
3		**Miscellaneous**					Solutions for Challenge Yourself	
4	**2 Integers and order of Operations Integers, Rational Numbers and Real Numbers**	**2.1 Negative Numbers (pp. 22 – 25)**	• Use negative numbers, rational numbers and real numbers in a real-world context • Represent real numbers on a number line and order the numbers • Explain what is an absolute and numerical value of an integer	Identify and use natural numbers and integers (positive, negative and zero) Use directed numbers in practical situations Order quantities by magnitude and demonstrate familiarity with the symbols =, ≠, <, >	Class Discussion – Use of Negative Numbers in the Real World (p. 22) Main Text (pp. 23 – 24) Thinking Time (p. 23)			Class Discussion – Use of Negative Numbers in the Real World (p. 22) Thinking Time (p. 23)
5		**2.2 Addition and Subtraction involving Negative Numbers (pp. 26 – 33)**	• Perform operations in real numbers, including using the calculator • Verify commutative, associative , and distributive properties	Use the four operations for calculations with whole numbers including correct ordering of operations and use of brackets.	Main Text (pp. 26 – 27) Class Discussion – Addition involving Negative Numbers (p. 27) Main Text (pp. 28 – 29) Class Discussion – Subtraction involving Negative Numbers (p. 29)	Main Text – 'Alternatively, you may visit http://www.shinglee.com.sg/ Student Resources/ to access the AlgeToolTM software.' (p. 30)		Class Discussion – Addition involving Negative Numbers (p. 27) Class Discussion – Subtraction involving Negative Numbers (p. 29)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
6		**2.3 Multiplication and Division involving Negative Numbers (pp. 33 – 38)**		Use of an electronic calculator efficiently Apply appropriate checks of accuracy	Main Text (pp. 33 – 35) Class Discussion – Multiplication involving Negative Numbers (p. 36) Main Text (pp. 36–37) Practise Now 7, 8, 8a, 8b			Class Discussion – Multiplication involving Negative Numbers (p. 36)
6		**2.4 Fractions and Decimals (pp. 39 – 44)**		Use the four operations for calculations with decimals, vulgar (and mixed) fractions including correct ordering of operations and use of brackets.				
6		**Miscellaneous**					Solutions for Challenge Yourself	
7	**3 Percentage**	**3.1 Introduction to Percentage (pp. 48 – 55)**	• Express a percentage as a fraction and vice versa • Express a percentage as a decimal and vice versa • Express one quantity as a percentage of another • Compare two quantities by percentage	Calculate a given percentage of a quantity Express one quantity as a percentage of another	Class Discussion – Percentage in Real Life (p. 48) Class Discussion – Finding the percentage of a quantity (p. 52)			Class Discussion – Percentage in Real Life (p. 48)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
7		**3.2 Percentage Change and Reverse Percentage (pp. 56 – 62)**	• Solve problems involving percentage change and reverse percentage	Calculate percentage increase or decrease Carry out calculations involving reverse percentages	Thinking Time (p. 61) Internet Resources (p. 61)	Internet Resources (p. 61)		Just for Fun (p. 58) Attention (p. 58) Thinking Time (p. 61)
7		**Miscellaneous**					Solutions for Challenge Yourself	
8	**4 Introduction to Sets**	**4.1 Introduction to Set Notations (p. 66 – 73)**	• Describe a set in words, list all the elements in a set, and describe the elements in a set • State and use the terms ‘set’, ‘element’, ‘equal sets’, ‘empty set’, ‘finite and infinite set’, ‘singleton set’ • State and use the terms ‘universal set’ • Use Venn diagrams to represent sets, including universal sets	Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets Definition of sets: e.g. $A = \{x : x \text{ is a natural number}\}$, $B = \{(x, y): y = mx + c\}$, $C = \{x : a < x < b\}$, $D = \{a, b, c, \ldots\}$	Class Discussion – Well-defined and Distinct Objects in a Set (p. 67) Thinking Time (p. 69)			Class Discussion – Well-defined and Distinct Objects in a Set (p. 67) Attention (p. 67) Thinking Time (p. 69)
8		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
9	**5** **Number Patterns and Algebraic Manipulation**	**5.1 Number Sequences** **(pp. 77 – 80)**	• Recognise simple patterns from various number sequences and determine the next few terms • Determine the next few terms and find a formula for the general term (nth term) of a number sequence • Solve problems involving number sequences and number patterns	Continue a given number sequence Recognise patterns in sequences and relationships between different sequences Generalise sequences as simple algebraic statements	Class Discussion –Number Sequence (p.77)			
9, 10		**5.2 Fundamental Algebra** **(pp. 81 – 90)**	• Use letters to represent numbers • Express basic arithmetical processes algebraically • Evaluate algebraic expressions • Add and subtract linear expressions	Use letters to express generalised numbers and express arithmetic processes algebraically Substitute numbers for words and letters in formulae	Class Discussion – Expressing Mathematical Relationships using Algebra (p. 82) Investigation – Comparison between Pairs of Expressions (p. 83)	Investigation – Comparison between Pairs of Expressions (p. 83) Journal Writing (p. 85)		Investigation – Comparison between Pairs of Expressions (p. 83) Journal Writing (p. 85)
11		**5.3 Expansion and Simplification of Linear Expressions** **(pp. 91 – 97)**	• Simplify linear expressions	Expand product of algebraic expressions	Main Text (pp. 91 – 96) Practise Now 7, 8 (p. 92) Practise Now 9 (p. 93) Practise Now 10 (p. 94) Class Discussion – Expanding an expression (p. 93) Thinking Time (p. 96)			Practise Now 7 (p. 92) Practise Now 8 (p. 92) Practise Now 9 (p. 93) Thinking Time (p. 96)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
11		**Miscellaneous**					Solutions for Challenge Yourself	
12, 13	**6** **Linear Equation**	**6.1 Linear Equations (pp. 101 – 110)**	• Explore the concepts of equation and inequality • Solve linear equations in one variable • Solve fractional equations that can be reduced to linear equations	Solve simple linear equations in one unknown Solve fractional equations with numerical and linear algebraic denominators	Main Text (pp. 101 – 108) Practise Now 1 (p. 102) Practise Now 2 (p. 103) Practise Now 3 (p. 104) Practise Now 4 (p. 105) Journal Writing (p. 105) Practise Now 5 (p. 107) Thinking Time (p. 107) Practise Now 6 (p. 108) Practise Now 7 (p. 109) Worked Example 2 (pp. 107 - 108) Worked Example 3 (p. 108)	Practise Now (p. 112) Practise Now (p. 113)		Main Text – 'From Table 6.1, discuss with your classmate what a linear equation is.' (p. 101) Journal Writing (p. 105) Thinking Time (p. 107)
12, 13		**6.2 Formulae (pp. 110 – 113)**	• Evaluate an unknown in a formula					Ex 6B Q 17(ii) (p. 113)
12, 13		**6.3 Applications of Linear Equations in Real-World Contexts (pp. 114 – 116)**	• Formulate linear equations to solve word problems					
12, 13		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
14	**7** **Rate and Ratio**	**7.1 Ratio** **(pp. 120 – 126)**	• Find ratios involving rational numbers • Find equivalent ratios • Solve problems involving ratio	Increase and decrease a quantity by a given ratio	Journal Writing (p. 124) Worked Example 2 (p. 121) Class Discussion – Making Sense of the Relationship between Ratios and Fractions (p. 122) Worked Example 3 (p. 123) Worked Example 4 (p. 123) Investigation – Golden Ratio (pp. 124 - 125) Performance Task (p. 125)			Journal Writing (p. 124) Performance Task (p. 125)
15		**7.2 Rate** **(pp. 127 – 130)**	• Solve problems involving rate		Thinking Time (p. 129)			Thinking Time (p. 129)
15		**Miscellaneous**					Solutions for Challenge Yourself	
16	**8** **Perimeter and Area of Plane Figures**	**8.1 Conversion of Units** **(p. 133)**	• Convert between cm^2 and m^2	Use current units of mass, length and area in practical situations and express quantities in terms of larger or smaller units	Class Discussion – International System of Units (p. 133)			Class Discussion –International System of Units (p. 133)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
16		**8.2 Perimeter and Area of Basic Plane Figures (pp. 134 – 138)**	• Find the perimeter and area of squares, rectangles, and triangles • Solve problems involving the perimeter and area • Find area of a path (inside or outside) a rectangle or a square	Solve problems involving the perimeter and area of a square, rectangle and triangle				
17		**8.3 Perimeter and Area of Parallelograms (pp. 139 – 142)**	• Find the perimeter and area of parallelograms	Solve problems involving the perimeter and area of a parallelogram	Investigation – Formula for Area of a Parallelogram (p. 140) Thinking Time (p. 140) Practise Now 5 (p. 141)	Thinking Time (p. 140)		Investigation – Formula for Area of a Parallelogram (p. 140) Thinking Time (p. 140)
17		**8.4 Perimeter and Area of a Trapezium (pp.143 – 146)**	• Find the perimeter and area of trapeziums	Solve problems involving the perimeter and area of a trapezium	Investigation – Formula for Area of a Trapezium (pp. 143 – 144) Practise Now 9 (p. 145) Thinking Time (p. 144)			Investigation – Formula for Area of a Trapezium (pp.143 - 144) Thinking Time (p. 144)
17		**Miscellaneous**					Solutions for Challenge Yourself	
18	**9 Volume and Surface Area of cubes and Cuboids**	**9.1 Conversion of Units (pp. 151– 152)**	• Convert between cm^3 and m^3	Use current units of volume and capacity in practical situations and express quantities in terms of larger or smaller units	Class Discussion – Measurements in Daily Lives (p. 151)			

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
18		**9.2 Recognise and Identify 3D shapes and their properties (p. 153)**	• Discuss properties of 3D shapes	Use and interpret nets to identify cubes and cuboids	Investigation – Cubes and Cuboids (pp. 153)			
19		**9.3 Surface Area of Cubes and Cuboids (pp. 154 – 158)**	• Find the volume and surface area of cubes and cuboids	Solve problems involving the surface area and volume of a cuboid	Class Discussion –Surface Area of Cubes and Cuboids (pp. 155 - 156)			Class Discussion – Surface Area of Cubes and Cuboids (pp. 155 – 156) Ex 9 A Q 17 (ii) (p. 158)
19		**Miscellaneous**					Solutions for Challenge Yourself	
20	**10 Basic Geometry**	**10.1 Points, Lines and Planes (pp. 162 – 164)**		Use and interpret the geometrical terms: point; line; plane	Thinking Time (p. 164)	Internet Resources (p. 164)		Thinking Time (p. 164)
20		**10.2 Angles (pp. 165 – 173)**	• Identify various types of angles • Solve problems involving angles on a straight line, angles at a point and vertically opposite angles	Use and interpret the geometrical terms: parallel; perpendicular; right angle, acute, obtuse and reflex angles, vertically opposite angles, complementary, and supplementary angles Calculate unknown angles and give simple explanations using the following geometrical properties: (a) angles at a point (b) angles at a point on a straight line and intersecting straight lines				Just for Fun (p. 165)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
21, 22		**10.3 Angles formed by Two Parallel Lines and a Transversal (pp. 173 – 180)**	• Solve problems involving angles formed by two parallel lines and a transversal, i.e. corresponding angles, alternate angles and interior angles	Calculate unknown angles and give simple explanations using angles formed within parallel lines	Investigation – Corresponding Angles, Alternate Angles and Interior Angles (p. 174)			Investigation – Corresponding Angles, Alternate Angles and Interior Angles (p. 174) Practise Now 6, 7 (p. 175) Practise Now 8 (p. 176) Practise Now 10 (p. 178) Ex 10B (p. 178)
21, 22		**Miscellaneous**					Solutions for Challenge Yourself	
23	**11 Geometrical Constructions**	**11.1 Introduction to Geometrical Constructions (pp. 186 – 187)**			Just for Fun (p.187)			
23		**11.2 Perpendicular Bisectors and Angle Bisectors (pp. 188 – 191)**	• Construct perpendicular bisectors and angle bisectors • Construct a perpendicular to a line from a point lying on it • Construct a perpendicular to a line from a point lying outside it • Apply properties of perpendicular bisectors and angle bisectors	Measure lines and angles Construct angle bisectors and perpendicular bisectors using a pair of compasses as necessary	Investigation – Property of a Perpendicular Bisector (p. 190) Investigation – Property of an Angle Bisector (p. 191)	Internet Resources (p. 190)		
24		**11.3 Construction of Angles (30, 45, 60, 75, 90, 105, 120) (pp. 192 – 196)**	• Construct angles	Construct angles, using a ruler and a pair of compasses only				

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
24		**11.4 Reflection (pp. 196 – 199)**	• Draw a line of reflection of figures	Draw reflection of figures using a grid paper Draw reflection of figures using a pair of compasses				
24		**Miscellaneous**					Solutions for Challenge Yourself	
25	**12 Symmetry**	**12.1 Line symmetry (pp. 202 – 211)**	• Identify line symmetry of plane figures	Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions	Investigation – Line Symmetry in Two Dimensions (pp. 202 - 203) Thinking Time (p. 203) Thinking Time (p. 204)			Thinking Time (p. 203) Thinking Time (p. 204)
25		**12.2 Rotational Symmetry in Plane Figures (pp. 212 – 216)**	• Identify rotational symmetry of plane figures		Investigation – Rotational Symmetry in Two Dimensions (pp. 212 - 213) Class Discussion – Line and Rotational Symmetry in Circles (p. 213)			Class Discussion – Line and Rotational Symmetry in Circles (p.213)
25		**Miscellaneous**					Solutions for Challenge Yourself	
26	**13 Statistical Data Handling**	**13.1 Introduction to Statistics (p. 219)**			Story Time (p. 219)			Story Time (p. 219)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
26		**13.2 Pictograms and Bar Graphs (pp. 219 – 225)**	• Collect, classify and tabulate data • Construct and interpret data from pictograms and multiple bar graphs	Collect, classify and tabulate statistical data Read, interpret and draw simple inferences from tables and statistical diagrams Construct and interpret pictograms and multiple bar charts	Thinking Time (p. 221)			Main Text – 'Two levels in the school are selected as the sample group for the survey conducted by the school canteen vendor. Are they representative of the entire school? Explain your answer.' (p. 219) Main Text – 'If the canteen vendor decides to sell three types of fruits to the students, which three should he choose? Explain your answer.' (p.220) Thinking Time (p. 221) Practise Now 2 Q (d) (i)(ii), (e) (p. 223) Ex 13 A Q 4(e), 5(iv), 6(iii) (p. 225)
27		**13.3 Pie Charts (pp. 226 – 228)**	• Construct and interpret data from pie charts	Construct and interpret pie charts	Main Text (pp. 226 – 227)			Practise Now 4 Q 2 (iii) (p. 228)
27		**13.4 Line Graphs (pp. 228 – 230)**	• Construct and interpret data from line graphs • Evaluate the purposes and appropriateness of the use of different statistical diagrams	Read, interpret and draw simple inferences from tables and statistical diagrams	Worked Example 2 Q (ii) (p. 229) Class Discussion – Comparison of Various Statistical Diagrams (p. 230)			Worked Example 2 Q (iv) (p. 229) Practise Now 5 Q (iv) (p. 230) Class Discussion – Comparison of Various Statistical Diagrams (p. 230)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
28		**13.5 Statistics in Real-World Contexts (pp. 231 – 232)**			Main Text (pp. 231 – 232) Performance Task (p.232)	Internet Resources (p. 232)		Performance Task (p. 232)
28		**13.6 Evaluation of Statistics (pp. 233 – 237)**	• Explain why some statistical information or diagrams can lead to a misinterpretation of data		Class Discussion – Evaluation of Statistics (pp. 233 – 234) Ex 13B Q 10 – 13 (pp. 236 – 237)			Class Discussion – Evaluation of Statistics (pp. 233 – 234) Ex 13B Q 8 (iv), 10 – 11, 12 (iii), 13 (pp. 236 – 237)
28		**Miscellaneous**					Solutions for Challenge Yourself	
28	**14 Averages of Statistical Data**	**14.1 Types of data (pp. 241 – 242)**	• Identify types of data	Distinguish between grouped and ungrouped data	Class Discussion — Evaluation of statistical diagram, descrete and continuous data (p. 242)			
29, 30		**14.2 Mean (pp. 243 - 248)**	• Find the mean of a set of data • Calculate an estimate for the mean	Calculate the mean for individual and discrete data Calculate an estimate of the mean for grouped and continuous data				
29, 30		**14.3 Median (pp. 248 – 251)**	• Find the median of a set of data • Find the class interval where the median lies	Calculate the median for individual and discrete data	Thinking Time (p.250) Class Discussion – Creating Sets of Data with Given Conditions (p. 250)			Thinking Time (p. 250)
29, 30		**14.4 Mode (pp. 252 – 254)**	• Find the mode of a set of data • State the modal class of a set of grouped data	Calculate the mode for individual and discrete data Identify the modal class from a grouped frequency distribution	Thinking Time (p.253)			

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
29, 30		**14.5 Mean, Median and Mode (p. 254 – 260)**	• Evaluate the purposes and appropriateness of the use of mean, median and mode	Distinguish between the purposes for which the mean, median and mode are used	Thinking Time (p.256) Class Discussion – Comparison of Mean, Median and Mode (p. 256)			Thinking Time (p. 253) Class Discussion – Comparison of Mean, Median and Mode (p. 256)
29, 30		**Miscellaneous**					Solutions for Challenge Yourself	
31	**15 Probability of Single Events**	**15.1 Introduction to Probability (p. 265)**	• Define probability as a measure of chance	Understand relative frequency as an estimate of probability	Thinking Time (p. 265)			Thinking Time (p. 265)
31		**15.2 Sample Space (pp. 266 – 268)**	• List the sample space of a probability experiment		Main Text (p. 266 – 268)			
32		**15.3 Probability of Single Events (pp. 269 – 278)**	• Find the probability of a single event	Calculate the probability of a single event as either a fraction or a decimal Understanding that the probability of an event occurring = 1 – probability of the event not occurring	Investigation – Tossing a Coin (pp. 269 – 270) Investigation – Rolling a Die (pp. 270 – 271) Thinking Time (p. 273) Performance Task (p. 273)	Internet Resources (p. 270)		Investigation – Tossing a Coin (pp.269 – 270) Investigation – Rolling a Die (pp. 270 - 271) Thinking Time (p. 273) Performance Task (p. 273) Just for Fun (p.275)
32		**Miscellaneous**					Solutions for Challenge Yourself	

Chapter 1 Primes, Highest Common Factor and Lowest Common Multiple

TEACHING NOTES

Suggested Approach

Students have learnt only whole numbers in primary school (they will only learn negative numbers and integers in Chapter 2). They have also learnt how to classify whole numbers into two groups, i.e. odd and even numbers. Teachers can introduce prime numbers as another way in which whole numbers can be classified (see Section 1.1). Traditionally, prime numbers apply to positive integers only, but the syllabus specifies whole numbers, which is not an issue since 0 is not a prime number. Teachers can also arouse students' interest in this topic by bringing in real-life applications (see chapter opener on page 2 of the textbook).

Section 1.1: Prime Numbers

Teachers can build upon prerequisites, namely, factors, to introduce prime numbers by classifying whole numbers according to the number of factors they have (see Investigation: Classification of Whole Numbers). Since the concept of 0 may not be easily understood, it is dealt with separately in the last question of the investigation. Regardless of whether 0 is classified in the same group as 1 or in a new fourth group, 0 and 1 are neither prime nor composite. Teachers are to take note that 1 is not a prime number 'by choice', or else the uniqueness of prime factorisation will fail (see Information on page 8 of the textbook). Also, 0 is not a composite number because it cannot be expressed as a product of prime factors unlike e.g. $40 = 2^3 \times 5$.

To make practice more interesting, a game is designed in Question 2 of Practise Now 1. Teachers can also tell students about the largest known prime number (there is no largest prime number since there are infinitely many primes) and an important real-life application of prime numbers in the encryption of computer data (see chapter opener and Investigation: Interesting Facts about Prime Numbers) in order to arouse their interest in this topic.

Section 1.3: Highest Common Factor and Lowest Common Multiple

Teachers can build upon prerequisites, namely, common factors and common multiples, to develop the concepts of Highest Common Factor (HCF) and Lowest Common Multiple (LCM) respectively. Since the listing method (see pages 12 and 14 of the textbook) is not an efficient method to find the HCF and the LCM of two or more numbers, there is a need to learn the prime factorisation method and the ladder method (see Methods 1 and 2 in Worked Example 8 and in Worked Example 10). However, when using the ladder method to find the LCM of two or three numbers (see Worked Examples 10 and 11), we stop dividing when there are no common prime factors between any two numbers.

Challenge Yourself

Some of the questions (e.g. Questions 2 and 3) are not easy for average students while others (e.g. Question 2) should be manageable if teachers guide them as follows:

Questions 2 and 3: Teachers can get students to try different numerical examples before looking for a pattern in order to generalise. In both questions, it is important that students know whether m and n are co-primes, i.e. HCF(m, n) = 1. If m and n are not co-primes, they can be built from the 'basic block' of $\frac{m}{\text{HCF}(m, n)}$ and $\frac{n}{\text{HCF}(m, n)}$, which are co-primes.

WORKED SOLUTIONS

Investigation (Classification of Whole Numbers)

1.

Number	Working	Factors
1	1 is divisible by 1 only.	1
2	2 = 1 × 2	1, 2
3	3 = 1 × 3	1, 3
4	4 = 1 × 4 = 2 × 2	1, 2, 4
5	5 = 1 × 5	1, 5
6	6 = 1 × 6 = 2 × 3	1, 2, 3, 6
7	7 = 1 × 7	1, 7
8	8 = 1 × 8 = 2 × 4	1, 2, 4, 8
9	9 = 1 × 9 = 3 × 3	1, 3, 9
10	10 = 1 × 10 = 2 × 5	1, 2, 5, 10
11	11 = 1 × 11	1, 11
12	12 = 1 × 12 = 2 × 6 = 3 × 4	1, 2, 3, 4, 6, 12
13	13 = 1 × 13	1, 13
14	14 = 1 × 14 = 2 × 7	1, 2, 7, 14
15	15 = 1 × 15 = 3 × 5	1, 3, 5, 15
16	16 = 1 × 16 = 2 × 8 = 4 × 4	1, 2, 4, 8, 16
17	17 = 1 × 17	1, 17
18	18 = 1 × 18 = 2 × 9 = 3 × 6	1, 2, 3, 6, 9, 18
19	19 = 1 × 19	1, 19
20	20 = 1 × 20 = 2 × 10 = 4 × 5	1, 2, 4, 5, 10, 20

Table 1.1

2. **Group A**: 1
   Group B: 2, 3, 5, 7, 11, 13, 17, 19
   Group C: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20
3. 0 is divisible by 1, 2, 3, 4, …
   0 has an infinite number of factors.

Thinking Time (Page 4)

1. A prime number is a whole number that has exactly 2 different factors, 1 and itself.
   A composite number is a whole number that has more than 2 different factors. A composite number has a finite number of factors.
   Since 0 has an infinite number of factors, it is neither a prime nor a composite number.
   Since 1 has exactly 1 factor, it is also neither a prime nor a composite number.
2. No, I do not agree with Maaz. Consider the numbers 0 and 1. They are neither prime numbers nor composite numbers.

Investigation (Sieve of Eratosthenes)

1.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

2. **(a)** The smallest prime number is 2.
   (b) The largest prime number less than or equal to 100 is 97.
   (c) There are 25 prime numbers which are less than or equal to 100.
   (d) No, not every odd number is a prime number, e.g. the number 9 is an odd number but it is a composite number.
   (e) No, not every even number is a composite number, e.g. the number 0 is an even number but it is neither a prime nor a composite number.
   (f) For a number greater than 5, if its last digit is 0, 2, 4, 6 or 8, then the number is a multiple of 2, thus it is a composite number; if its last digit is 0 or 5, then the number is a multiple of 5, thus it is a composite number. Hence, for a prime number greater than 5, its last digit can only be 1, 3, 7 or 9.

Journal Writing (Page 5)

1. Yes, the product of two prime numbers can be an odd number, e.g. the product of the two prime numbers 3 and 5 is the odd number 15.
2. Yes, the product of two prime numbers can be an even number, e.g. the product of the two prime numbers 2 and 3 is the even number 6.
3. No, the product of two prime numbers P_1 and P_2 cannot be a prime number since P_1P_2 has at least 3 distinct factors, i.e. 1, P_1 and P_1P_2.

Investigation (Interesting Facts about Prime Numbers)

The 1 000 000$^{\text{th}}$ prime number is 15 485 863.
The last digit of the largest known prime number is 1.

Thinking Time (Page 8)

The index notation is useful for writing complicated and repetitive expressions in a more compact form, e.g. we can write
$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ as 3^8.

Practise Now 1

1. 537 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 537 is $5 + 3 + 7 = 15$ which is divisible by 3, therefore 537 is divisible by 3 (divisibility test for 3).
$\therefore$ 537 is a composite number.

59 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 59 is $5 + 9 = 14$ which is not divisible by 3, then 59 is not divisible by 3.
The last digit of 59 is neither 0 nor 5, so 59 is not divisible by 5.
A calculator may be used to test whether 59 is divisible by prime numbers more than 5.
Since 59 is not divisible by any prime numbers less than 59, then 59 is a prime number.

2.

135	49	183	147	93	121	236
201	261	150	11	131	5	89
291	117	153	End	57	0	61
192	231	27	1	111	100	149
17	103	43	7	127	51	53
83	33	32	105	29	71	37

Start

Practise Now 2

1. Since 31 is a prime number, then 1 and 31 are its only two factors.
It does not matter whether p or q is 1 or 31 as we only want to find the value of $p + q$.
$\therefore p + q = 1 + 31 = 32$

2. Since $n \times (n + 28)$ is a prime number, then n and $n + 28$ are its only two factors.
Since 1 has to be one of its two factors, then $n = 1$.
$\therefore n \times (n + 28) = 1 \times (1 + 28)$
$= 1 \times 29$
$= 29$

Practise Now 3

1. $126 = 2 \times 3^2 \times 7$
2. $539 = 7^2 \times 11$

Practise Now 4

$\sqrt{2013} = 44.9$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{2013}$ is 43.
2013 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 2013 is $2 + 1 + 3 = 6$ which is divisible by 3, therefore 2013 is divisible by 3 (divisibility test for 3).
$\therefore$ 2013 is a composite number.

$\sqrt{2017} = 44.9$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{2017}$ is 43.
Since 2017 is not divisible by any of the prime numbers 2, 3, 5, 7, …, 43, then 2017 is a prime number.

Practise Now 5

1. **Method 1:**
$56 = 2^3 \times 7$
$84 = 2^2 \times 3 \times 7$
HCF of 56 and 84 $= 2^2 \times 7$
$= 28$

Method 2:

$$\begin{array}{r|l} 2 & 56,\ 84 \\ \hline 2 & 28,\ 42 \\ \hline 7 & 14,\ 21 \\ \hline & 2,\ 3 \end{array}$$

HCF of 56 and 84 $= 2 \times 2 \times 7$
$= 28$

2. $28 = 2^2 \times 7$
$70 = 2 \times 5 \times 7$
Largest whole number which is a factor of both 28 and 70
= HCF of 28 and 70
$= 2 \times 7$
$= 14$

3. Greatest whole number that will divide both 504 and 588 exactly
= HCF of 504 and 588
$= 2^2 \times 3 \times 7$
$= 84$

Practise Now 6

$90 = 2 \times 3^2 \times 5$
$135 = 3^3 \times 5$
$270 = 2 \times 3^3 \times 5$
HCF of 90, 135 and 270 $= 3^2 \times 5$
$= 45$

Practise Now 7

1. **Method 1:**
   $24 = 2^3 \times 3$
   $90 = 2 \times 3^2 \times 5$
   LCM of 24 and 90 $= 2^3 \times 3^2 \times 5$
   $= 360$

   Method 2:

   $$\begin{array}{r|l} 2 & 24, 90 \\ \hline 3 & 12, 45 \\ \hline & \quad 4, 15 \end{array}$$

   HCF of 24 and 90 $= 2 \times 3 \times 4 \times 15$
   $= 360$
2. Smallest whole number that is divisible by both 120 and 126
   = LCM of 120 and 126
   $= 2^3 \times 3^2 \times 5 \times 7$
   $= 2520$
3. $6 = 2 \times 3$
   $24 = 2^3 \times 3$
   Smallest value of $n = 2^3$
   $= 8$

Practise Now 8

$9 = 3^2$
$30 = 2 \times 3 \times 5$
$108 = 2^2 \times 3^3$
LCM of 9, 30 and 108 $= 2^2 \times 3^3 \times 5$
$= 540$

Practise Now 9

1. $15 = 3 \times 5$
   $16 = 2^4$
   $36 = 2^2 \times 3^2$
   LCM of 15, 16 and 36 $= 2^4 \times 3^2 \times 5$
   $= 720$
   720 minutes = 12 hours
   $\therefore$ The three bells will next toll together at 2.00 a.m.
2. **(i)** $140 = 2^2 \times 5 \times 7$
   $168 = 2^3 \times 3 \times 7$
   $210 = 2 \times 3 \times 5 \times 7$
   HCF of 140, 168 and 210 $= 2 \times 7$
   $= 14$
   Greatest possible length of each of the smaller pieces of rope
   = 14 cm

   (ii) Number of smallest pieces of rope he can get altogether
   $= \frac{140}{14} + \frac{168}{14} + \frac{210}{14}$
   $= 10 + 12 + 15$
   $= 37$

Exercise 1A

1. **(a)** 87 is an odd number, so it is not divisible by 2.
   Since the sum of the digits of 87 is $8 + 7 = 15$ which is divisible by 3, therefore 87 is divisible by 3 (divisibility test for 3).
   $\therefore$ 87 is a composite number.

   (b) 67 is an odd number, so it is not divisible by 2.
   Since the sum of the digits of 67 is $6 + 7 = 13$ which is not divisible by 3, then 67 is not divisible by 3.
   The last digit of 67 is neither 0 nor 5, so 67 is not divisible by 5.
   A calculator may be used to test whether 67 is divisible by prime numbers more than 5.
   Since 67 is not divisible by any prime numbers less than 67, then 67 is a prime number.

   (c) 73 is an odd number, so it is not divisible by 2.
   Since the sum of the digits of 73 is $7 + 3 = 10$ which is not divisible by 3, then 73 is not divisible by 3.
   The last digit of 73 is neither 0 nor 5, so 73 is not divisible by 5.
   A calculator may be used to test whether 73 is divisible by prime numbers more than 5.
   Since 73 is not divisible by any prime numbers less than 73, then 73 is a prime number.

   (d) 91 is an odd number, so it is not divisible by 2.
   Since the sum of the digits of 91 is $9 + 1 = 10$ which is not divisible by 3, then 91 is not divisible by 3.
   The last digit of 91 is neither 0 nor 5, so 91 is not divisible by 5.
   A calculator may be used to test whether 91 is divisible by prime numbers more than 5.
   Since 91 is divisible by 7, therefore 91 is a composite number.
2. **(a)** 15, 30, 45, 60, 75 **(b)** 28, 56, 845, 112, 140
   (c) 53, 106, 159, 212, 265 **(d)** 60, 120, 180, 240, 300
   (e) 33, 66, 99, 132, 165 **(f)** 41, 82, 123, 164, 205
3. **(a)** $72 = 2^3 \times 3^2$ **(b)** $187 = 11 \times 17$
   (c) $336 = 2^4 \times 3 \times 7$ **(d)** $630 = 2 \times 3^2 \times 5 \times 7$
4. **(a)** $42 \times 42 = 1764$ **(b)** $51 \times 51 = 2601$
   (c) $48 \times 48 = 2304$ **(d)** $92 \times 92 = 8464$
   (e) $86 \times 86 = 7396$ **(f)** $75 \times 75 = 5626$
5. $?^2 + 13^2 = 794$
   $?^2 + 169 = 794$
   $?^2 = 794 - 169$
   $?^2 = 625$
   $? \times ? = 625$
   $25 \times 25 = 625$
   Therefore, the other number is 25.
6. $65^2 - 45^2$
   $= 4225 - 2025$
   $= 2200$
7. **(a)** $\sqrt{66} \approx \sqrt{64}$
   $= 8$

(b) $\sqrt{80} \approx \sqrt{81}$
$= 9$

(c) $\sqrt[3]{218} \approx \sqrt[3]{216}$
$= 6$

(d) $\sqrt[3]{730} \approx \sqrt[3]{729}$
$= 9$

8. **(a)** $7^2 - \sqrt{361} + 21^3 = 9291$

(b) $\dfrac{\sqrt{555} + 5^2}{2^3 \times \sqrt[3]{222}} = 1.0024$ (to 4 d.p.)

(c) $\sqrt{4^3 + \sqrt[3]{4913}} = 9$

9. We know that,

$9^2 = 9 \times 9 = 81$

$10^2 = 10 \times 10 = 100$

90 lies between 81 and 100.

90 is not a perfect square.

10. Length of a square is 15 cm.

It's area is the square of its length.

$l^2 = 15 \times 15\ \text{cm}^2$

$= 225\ \text{cm}^2$

11. **(a)** $53 \times 53 = 2809$
2809 is a perfect square.

(b) 1155 is not a perfect square.

(c) 2021 is not a perfect square.

(d) $41 \times 41 = 1681$
1681 is a perfect squre.

12. Since 37 is a prime number, then 1 and 37 are its only two factors.
It does not matter whether p or q is 1 or 37 as we only want to find the value of $p + q$.
$\therefore\ p + q = 1 + 37 = 38$

13. Since $n \times (n + 42)$ is a prime number, then n and $n + 42$ are its only two factors.
Since 1 has to be one of its two factors, then $n = 1$.
$\therefore\ n \times (n + 42) = 1 \times (1 + 42)$
$= 1 \times 43$
$= 43$

Exercise 1B

1. **(a)** $12 = 2^2 \times 3$
$30 = 2 \times 3 \times 5$
HCF of 12 and 30 $= 2 \times 3$
$= 6$

(b) $84 = 2^2 \times 3 \times 7$
$156 = 2^2 \times 3 \times 13$
HCF of 84 and 156 $= 2^2 \times 3$
$= 12$

(c) $15 = 3 \times 5$
$60 = 2^2 \times 3 \times 5$
$75 = 3 \times 5^2$
HCF of 15, 60 and 75 $= 3 \times 5$
$= 15$

(d) $77 = 7 \times 11$
$91 = 7 \times 13$
$143 = 11 \times 13$
HCF of 77, 91 and 143 = 1

2. **(a)** $24 = 2^3 \times 3$
$30 = 2 \times 3 \times 5$
LCM of 24 and 30 $= 2^3 \times 3 \times 5$
$= 120$

(b) $42 = 2 \times 3 \times 7$
$462 = 2 \times 3 \times 7 \times 11$
LCM of 42 and 462 $= 2 \times 3 \times 7 \times 11$
$= 462$

(c) $12 = 2^2 \times 3$
$18 = 2 \times 3^2$
$81 = 3^4$
LCM of 12, 18 and 81 $= 2^2 \times 3^4$
$= 324$

(d) $63 = 3^2 \times 7$
$80 = 2^4 \times 5$
$102 = 2 \times 3 \times 17$
LCM of 63, 80 and 102 $= 2^4 \times 3^2 \times 5 \times 7 \times 17$
$= 85\ 680$

3. $42 = 2 \times 3 \times 7$
$98 = 2 \times 7^2$
Largest whole number which is a factor of both 42 and 98
= HCF of 42 and 98
$= 2 \times 7$
$= 14$

4. Greatest whole number that will divide both 792 and 990 exactly
= HCF of 792 and 990
$= 2 \times 3^2 \times 11$
$= 198$

5. Smallest whole number that is divisble by both 176 and 342
= LCM of 176 and 342
$= 2^4 \times 3^2 \times 11 \times 19$
$= 30\ 096$

6. $15 = 3 \times 5$
$45 = 3^2 \times 5$
Smallest value of $n = 3^2$
$= 9$

7. **(i)** $171 = 3^2 \times 19$
$63 = 3^2 \times 7$
$27 = 3^3$
HCF of 171, 63 and 27 $= 3^2$
$= 9$
Largest number of gift bags that can be packed = 9

(ii) Number of pens in a gift bag $= 171 \div 9$
$= 19$

Number of pencils in a gift bag = $63 \div 9$
$= 7$
Number of erasers in a gift bag = $27 \div 9$
$= 3$

8. **(i)** $60 = 2^2 \times 3 \times 5$
$80 = 2^4 \times 5$
LCM of 60 and 80 = $2^4 \times 3 \times 5$
$= 240$
It will take 240 s for both cars to be back at the starting point at the same time.

(ii) 5×240 s = 1200 s
= 20 minutes
It will take 20 minutes for the faster car to be 5 laps ahead of the slower car.

9. **(a)** True.
If 6 is a factor of a whole number n, then $n = 6k$ for some whole number k.
We have $n = 6k = 2(3k)$. Since $3k$ is a whole number, then 2 is a factor of n.
We also have $n = 6k = 3(2k)$. Since $2k$ is a whole number, then 3 is a factor of n.

(b) True. Since 2 and 3 are distinct prime factors of the whole number, then the prime factorisation of the whole number will contain both of these prime factors.

(c) False, e.g. 2 and 4 are factors of 4, but 8 is not a factor of 4.

(d) True. If f is a factor of n, then $n = fk$ for some whole number k. Thus $\frac{n}{f} = k$ is a whole number. Since n can be written as a product of the whole numbers $\frac{n}{f}$ and f, then $\frac{n}{f}$ is a factor of n.

(e) True. Since h is a factor of both p and q, then both p and q are divisible by h.

10. $9 = 3^2$
$12 = 2^2 \times 3$
$252 = 2^2 \times 3^2 \times 7$
Possible values of n = 7, 3×7 or $3^2 \times 7$
= 7, 21 or 63

11. **(a)** True. If 6 is a multiple of a whole number n, then $6 = nk$ for some whole number k. We have $12 = 2nk = n(2k)$. Since $2k$ is a whole number, then 12 is a multiple of n.

(b) False, e.g. 12 is a multiple of 4, but 6 is not a multiple of 4.

(c) True. If 18 is a multiple of a whole number n, then $18 = nk$ for some whole number k. Thus $\frac{18}{n} = k$ is a whole number, i.e. 18 is divisible by n.

(d) True. Since m is a multiple of p, by the same reasoning as in **(c)**, then m is divisible by p. Similarly, m is divisible by q.

12. **(i)** $64 = 2^6$
$48 = 2^4 \times 3$
HCF of 64 and 48 = 2^4
$= 16$
Length of each square = 16 cm

(ii) Number of squares that can be cut altogether = $\frac{64}{16} \times \frac{48}{16}$
$= 4 \times 3$
$= 12$

13. **(i)** Let the number of boys in the class be n.
Then $15 \times n = 3 \times 5 \times n$ is divisible by $21 = 3 \times 7$.
Thus the possible values of n are multiples of 7.
Hence, $n = 14$ since $14 + 20 = 34$ students is the only possibility where the number of students in the class is between 30 and 40.
∴ Number of students in the class = 34

(ii) Number of chocolate bars their form teacher receive
$= \frac{15 \times 14}{21}$
$= 10$

14. **(i)** $126 = 2 \times 3^2 \times 7$
$108 = 2^2 \times 3^3$
HCF of 126 and 108 = 2×3^3
$= 18$
Length of each square = 18 cm
Least number of square patterns that could be formed on the sheet of paper
$= \frac{126}{18} \times \frac{108}{18}$
$= 7 \times 6$
$= 42$

(ii) To fit the sheet of paper perfectly, the patterns can be rectangular, triangular or trapeziums with two right angles, etc.

15. **(i)** $45 = 3^2 \times 5$
$42 = 2 \times 3 \times 7$
LCM of 45 and 42 = $2 \times 3^2 \times 5 \times 7$
$= 630$
Number of patterns needed to form the smallest square
$= \frac{630}{45} \times \frac{630}{42}$
$= 14 \times 15$
$= 210$

(ii) 630 mm = 0.63 m
Area of smallest square that can be formed = 0.63^2
$= 0.3969$ m^2
By trial and error,
Area of largest square that can be formed
$= 0.3969 \times 2^2$
$= 1.5876$ m^2 < 1.6 m^2
∴ Length of largest square that can be formed = $\sqrt{1.5876}$
= 1.26 m

Review Exercise 1

1. $6 = 2 \times 3$

$12 = 2^2 \times 3$

$660 = 2^2 \times 3 \times 5 \times 11$

Possible values of $n = 5 \times 11, 2 \times 5 \times 11, 3 \times 5 \times 11,$
$2^2 \times 5 \times 11, 2 \times 3 \times 5 \times 11$
or $2^2 \times 3 \times 5 \times 11$
$= 55, 110, 165, 220, 330$ or 660

2. **(i)** $108 = 2^2 \times 3^3$

$81 = 3^4$

$54 = 2 \times 3^3$

HCF of 108, 81 and 54 $= 3^3$
$= 27$

Largest number of baskets that can be packed = 27

(ii) Number of stalks of roses in a basket $= 108 \div 27$
$= 4$

Number of stalks of lilies in a basket $= 81 \div 27$
$= 3$

Number of stalks of orchids in a basket $= 54 \div 27$
$= 2$

3. Time taken for Hussain to run 1 round = 360 s
= 6 minutes

Time taken for Beena to cycle 1 round $= 4 \div 2$
= 2 minutes

$18 = 2 \times 3^2$

$6 = 2 \times 3$

$2 = 2$

LCM of 18, 6 and 2 $= 2 \times 3^2$
$= 18$

All three of them will next meet at 6.03 p.m.

4. **(i)** By counting, they will next have the same day off on 7 May.

(ii) $4 = 2^2$

$6 = 2 \times 3$

LCM of 4 and 6 $= 2^2 \times 3$
$= 12$

Subsequently, they will have the same day off every 12 days.

Challenge Yourself

1. **(i)** $120 = 2^3 \times 3 \times 5$

$126 = 2 \times 3^2 \times 7$

HCF of 120 and 126 $= 2 \times 3$
$= 6$

LCM of 120 and 126 $= 2^3 \times 3^2 \times 5 \times 7$
$= 2520$

(ii) HCF × LCM $= 6 \times 2520$
$= 15\,120$
$= 120 \times 126$ (Shown)

$120 = \mathbf{2^3} \times \mathbf{3} \times \mathbf{5}$

$126 = \mathbf{2} \times \mathbf{3^2} \times \mathbf{7}$

To obtain the **HCF** of 120 and 126, we choose the power of each of the common prime factors with the smaller index, i.e. **2** and **3**.

On the other hand, to obtain the **LCM** of 120 and 126, we choose the power of each of the common prime factors with the higher index, i.e. $\mathbf{2^3}$ and $\mathbf{3^2}$, and the remaining factors, i.e. **5** and **7**.

Since each term in the prime factorisation of 120 and 126 is used to find either their **HCF** or their **LCM**, the product of the **HCF** and **LCM** of 120 and 126 is equal to the product of 120 and 126.

(iii) Yes, the result in **(ii)** can be generalised for any two numbers.

Proof:

Consider two numbers x and y.

Then $x = \text{HCF} \times p$, — (1)

$y = \text{HCF} \times q$, — (2)

where the HCF of p and q is 1.

$(1) \times q$: $x \times q = \text{HCF} \times p \times q$ — (3)

$(2) \times p$: $y \times p = \text{HCF} \times p \times q$ — (4)

$(3) \times (4)$: $x \times y \times p \times q = \text{HCF} \times p \times q \times \text{HCF} \times p \times q$

$x \times y = \text{HCF} \times \text{HCF} \times p \times q$

Since the HCF of p and q is 1, we cannot take out a factor greater than 1 in the product $p \times q$, thus $\text{HCF} \times p \times q = \text{LCM}$.

$\therefore x \times y = \text{HCF} \times \text{LCM}$

(iv) No, the result in **(ii)** cannot be generalised for any three numbers.

For example, consider the numbers 10, 20 and 25.

$10 = 2 \times 5$

$20 = 2^2 \times 5$

$25 = 5^2$

HCF of 10, 20 and 25 = 5

LCM of 10, 20 and 25 $= 2^2 \times 5^2$
$= 100$

HCF × LCM $= 5 \times 100$
$= 500$
$\neq 10 \times 20 \times 25$

2. Number of squares passed through by a diagonal of a m-by-n rectangle

$= m + n - \text{HCF}(m, n)$

3. **(i)** Fraction of a sausage each person gets $= \frac{12}{18}$
$= \frac{2}{3}$

$\therefore$ Least number of cuts required = 12

(ii) Least number of cuts required $= n - \text{HCF}(m, n)$

Chapter 2 Integers and Order of Operations

TEACHING NOTES

Suggested Approach

Although the concept of negative numbers is new to most students as they have not learnt this in primary school, they do encounter negative numbers in their daily lives, e.g. in weather forecasts. Therefore, teachers can get students to discuss examples of the use of negative numbers in the real world to bring across the idea of negative numbers (see Class Discussion: Uses of Negative Numbers in the Real World). The learning experiences in the new syllabus specify the concrete objects, such as use of algebra discs. In this chapter, only number discs (or counters) showing the numbers 1 and –1 are needed. Since many Grade 6 students are still in the concrete operational stage (according to Piaget), the use of algebra discs can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use algebra discs in examinations, and partly because they cannot use algebra discs to add or subtract large negative integers, and decimals (see Section 2.2).

Section 2.1: Negative Numbers

Teachers should teach students to read the negative number –2 as negative 2, not minus 2 ('negative' is a state while 'minus' is an operation). For example, if you have \$5 and you owe your friend \$2, how much do you have left? Since nothing is mentioned about you returning money to your friend, you have \$5 left. Thus \$2 is a state of owing money. However, if you return \$2 to your friend, you have \$5 + (–\$2) = \$5 – \$2 = \$3 left, i.e. 5 minus 2 is an operation of returning money.

Students should also learn about the absolute value of a negative number (see page 24 of the textbook) because they will need it in Section 2.2.

In primary school, students have only learnt the terms 'less than' and 'more than', so there is a need to teach them how to use the symbols '<' and '>' when comparing numbers. It is not necessary to teach them about 'less than or equal to' and 'more than or equal to' now.

Section 2.2: Addition and Subtraction involving Negative Numbers

Algebra discs cannot be used to add or subtract large negative integers, and decimals, so there is a need to help students consolidate what they have learnt in the class discussions on pages 27 and 29 of the textbook by moving away from the 'concrete' to the following two key 'abstract' concepts:

Key Concept 1: Adding a negative number is the same as subtracting the absolute value of the number, e.g. $5 + (-2) = 5 - 2$.

Key Concept 2: Subtracting a negative number is the same as adding the absolute value of the number, e.g. $5 - (-2) = 5 + 2$.

To make the key concepts less abstract, numerical examples are used. Do not use algebra now because students are still unfamiliar with algebra even though they have learnt some basic algebra in primary school. Avoid teaching students '$- \times - = +$' now because the idea behind $5 - (-2)$ is subtraction, not multiplication. To make practice more interesting, a puzzle is designed on page 31 of the textbook.

Section 2.3: Multiplication and Division involving Negative Numbers

The idea of flipping over a disc to obtain the negative of a number, e.g. $-(-3) = 3$, is important in teaching multiplication involving negative numbers. Since algebra discs cannot be used to teach division involving negative numbers, another method is adopted (see page 36 of the textbook).

Section 2.4: Fractions and Decimals

Students will use their prior knowledge about integers and order of operations to carry out mathematical sums relating to fractions and decimals. The teacher may use Worked Examples 4-9 to support students in learning.

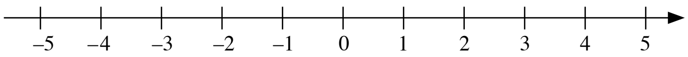
WORKED SOLUTIONS

Class Discussion (Uses of Negative Numbers in the Real World)

- One of the most common uses of negative numbers is in the measurement of temperature, where negative numbers are used to show temperatures below the freezing point of water, i.e. 0 °C. Absolute zero, defined as 0 Kelvin, is the theoretical lowest possible temperature. 0 Kelvin is equivalent to a temperature of –273.15 °C, therefore the theoretical lowest possible temperature is 273.15 °C below 0 °C.
- The elevation of a location commonly refers to its height with reference to Earth's sea level and can be represented by a positive or a negative number. Given a point with an elevation of –200 m, we can deduce that the point is 200 m below sea level. The lowest elevation on Earth that is on dry land is the Dead Sea shore in Asia with an elevation of –423 m, i.e. the shore of the Dead Sea is 423 m below sea level.
- Negative numbers are also used to tell time zones, which are based on Greenwich Mean Time (GMT). A country which is in the time zone of GMT –2 means that the time in that country is 2 hours behind the GMT. For example, Honolulu, Hawaii is in the time zone of GMT –10, while Liverpool, United Kingdom is in the time zone of GMT 0, therefore when it is 10 a.m. in Liverpool, it is 12 midnight in Honolulu.
- Latitude and longitude are a set of coordinates that allow for the specification of a geographical location on the Earth's surface and can be represented by positive and/or negative numbers. The latitude of a point is determined with reference to the equatorial plane; the North Pole has a latitude of +90°, which means that it is 90° north of the equator while the South Pole has a latitude of –90°, which means that it is 90° south of the equator. The longitude of a point gives its east-west position relative to the Prime Meridian (longitude 0°); a location with a longitude of +50° means that it is 50° east of the Prime Meridian while a location with a longitude of –50° means that it is 50° west of the Prime Meridian. The latitude and longitude of Rio Grande, Mexico are approximately –32° and –52° respectively, which means that it is 32° south of the equator and 52° west of the Prime Meridian.
- The use of negative numbers can also be seen in scoring systems, such as in golf. Each hole has a par score, which indicates the number of strokes required and a golfer's score for that hole is calculated based on the number of strokes played. A score of +3 on a hole shows that the golfer played three strokes above par, while a score of –3 on a hole shows that the golfer played three strokes under par.

Teachers may wish to note that the list is not exhaustive.

Thinking Time (Page 23)



(a) Since –3 is on the left of 2, we say '–3 is less than 2' and we write '$-3 < 2$'.

(b) Since –3 is on the right of –5, we say '–3 is more than –5' and we write '$-3 > -5$'.

Class Discussion (Addition involving Negative Numbers)

Part I

1. **(a)** $7 + (-3) = 4$
(b) $6 + (-4) = 2$

2. **(a)** $(-7) + 3 = -4$
(b) $(-6) + 4 = -2$

3. **(a)** $(-7) + (-3) = -10$
(b) $(-6) + (-4) = -10$

Note:

- If we add a positive number and a negative number,
  (i) we take the *difference* between the absolute values of the two numbers, and
  (ii) the sign of the answer follows the sign of the number with the greater absolute value,
  e.g. $5 + (-2) = 3$ and $(-5) + 2 = -3$.
- If we add two negative numbers,
  (i) we take the *sum* of the absolute values of the two numbers, and
  (ii) the answer is negative,
  e.g. $(-5) + (-2) = -7$.

Class Discussion (Subtraction involving Negative Numbers)

Part I

1. **(a)** $7 - (-3) = 7 + 3$
$= 10$
(b) $6 - (-4) = 6 + 4$
$= 10$

2. **(a)** $(-7) - 3 = (-7) + (-3)$
$= -10$
(b) $(-6) - 4 = (-6) + (-4)$
$= -10$

3. **(a)** $(-7) - (-3) = (-7) + 3$
$= -4$
(b) $(-4) - (-6) = (-4) + 6$
$= 2$

4. **(a)** $3 - 7 = 3 + (-7)$
$= -4$
(b) $4 - 6 = 4 + (-6)$
$= -2$

Note:

- If we take the difference of a positive number and a negative number,
  (i) we *add* the absolute values of the two numbers, and
  (ii) the sign of the answer follows the sign of the number with the greater absolute value,
  e.g. 5 – (–2) = 7 and (–5) – 2 = –7.
- If we take the difference of two negative numbers or two positive numbers,
  (i) we take the *difference* between the absolute values of the two numbers, and
  (ii) the sign of the answer depends on whether the first number is greater than or smaller than the second number,
  e.g. (–5) – (–2) = –3 but (–2) – (–5) = 3;
  2 – 5 = –3 but 5 – 2 = 3.

Class Discussion (Multiplication involving Negative Numbers)

Part I

1. **(a)** 1 × (–4) = –4
(b) 2 × (–4) = –8
(c) 3 × (–4) = –12

2. **(a)** (–1) × 4 = –4
(b) (–2) × 4 = –8
(c) (–3) × 4 = –12

3. **(a)** (–1) × (–4) = 4
(b) (–2) × (–4) = 8
(c) (–3) × (–4) = 12

Note: In general,

positive number × *negative* number = *negative* number,
negative number × *positive* number = *negative* number,
negative number × *negative* number = *positive* number.

Practise Now 1

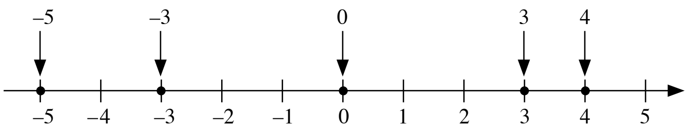
1. **(i)** 2013, 6
(ii) –5, –17
(iii) 2013, 1.666, $\frac{3}{4}$, 6
(iv) –5, $-\frac{1}{2}$, –3.8, –17, $-\frac{2}{3}$

2. **(a)** –43.6 °C
(b) –423 m
(c) –1
(d) –$10 000

Practise Now 2

1. **(b)** < **(c)** > **(d)** >

2.



∴ –5, –3, –0, 3, 4

Practise Now 3

(a) 9 + (–2) = 7
(b) –7 + 4 = –3
(c) 3 + (–5) = –2
(d) –6 + (–8) = –14
(e) 27 + (–13) = 14
(f) –25 + 11 = –14
(g) 14 + (–16) = –2
(h) –12 + (–15) = –27

Practise Now 4

Temperature in the morning = –8 °C + 2 °C
= –6 °C

Practise Now 5

(a) 9 – (–2) = 9 + 2
= 11
(b) –7 – 4 = –11
(c) –3 – (–5) = –3 + 5
= 2
(d) –8 – (–6) = –8 + 6
= –2
(e) 4 – 8 = –4
(f) 27 – (–13) = 27 + 13
= 40
(g) –25 – 11 = –36
(h) –14 – (–16) = –14 + 16
= 2
(i) –15 – (–12) = –15 + 12
= –3
(j) 10 – 28 = –18

Practise Now 6

1. Point *A* shows –5 °C.
Point *B* shows 23 °C.
Difference in temperature = 23 °C – (–5 °C)
= 23 °C + 5 °C
= 28 °C

2. Altitude at $D = -165$ m

Difference in altitude $= 314 \text{ m} - (-165 \text{ m})$
$= 314 \text{ m} + 165 \text{ m}$
$= 479 \text{ m}$

Practise Now 7

(a) $2 \times (-6) = -12$
(b) $-5 \times 4 = -20$
(c) $-1 \times (-8) = 8$
(d) $-3 \times (-7) = 21$
(e) $-(-10) = 10$
(f) $-9(-2) = 18$
(g) $15 \times (-2) = -30$
(h) $-3 \times 12 = -36$
(i) $-4 \times (-10) = 40$
(j) $-2(-100) = 200$

Practise Now 8

(a) $-8 \div 2 = -4$
(b) $15 \div (-3) = -5$
(c) $-8 \div (-4) = 2$
(d) $\frac{-6}{3} = -2$
(e) $\frac{20}{-5} = -4$
(f) $\frac{-12}{-3} = 4$

Practise Now 9a

(a)
$$\begin{aligned} -3 \times (15 - 7 + 2) &= -3 \times (8 + 2) \\ &= -3 \times 10 \\ &= -30 \end{aligned}$$

(b)
$$\begin{aligned} 4^3 - 7 \times [16 - (\sqrt[3]{64} - 5)] &= 64 - 7 \times [16 - (4 - 5)] \\ &= 64 - 7 \times [16 - (-1)] \\ &= 64 - 7 \times (16 + 1) \\ &= 64 - 7 \times 17 \\ &= 64 - 119 \\ &= -55 \end{aligned}$$

Practise Now 9b

(a) $-3 \times (15 - 7 + 2) = -30$
(b) $4^3 - 7 \times [16 - (\sqrt[3]{64} - 5)] = -55$

Practise Now 10

$$\begin{aligned} 7\frac{1}{2} + -3\frac{3}{5} &= 7\frac{1}{2} - 3\frac{3}{5} \\ &= 7\frac{5}{10} - 3\frac{6}{10} \\ &= 6 + \frac{10}{10} + \frac{5}{10} - 3\frac{6}{10} \\ &= 3\frac{9}{10} \end{aligned}$$

Practise Now 11

(a)
$$\begin{aligned} 2\frac{2}{3} \times \frac{9}{4} &= \frac{{}^{2}\cancel{8}}{{}_{1}\cancel{3}} \times \frac{\cancel{9}^{3}}{\cancel{4}_{1}} \\ &= 2 \times 3 \\ &= 6 \end{aligned}$$

(b)
$$\begin{aligned} 4\frac{1}{6} \div \frac{5}{2} &= \frac{25}{6} \div \frac{5}{2} \\ &= \frac{{}^{5}\cancel{25}}{{}_{3}\cancel{6}} \times \frac{\cancel{2}^{1}}{\cancel{5}_{1}} \\ &= \frac{5}{3} \\ &= 1\frac{2}{3} \end{aligned}$$

Practise Now 12a

$$\begin{aligned} 5\frac{1}{4} \div -2\frac{4}{5} &= \frac{21}{4} \div -\frac{14}{5} \\ &= \frac{{}^{3}\cancel{21}}{4} \times -\frac{5}{\cancel{14}_{2}} \\ &= -\frac{15}{8} \\ &= -1\frac{7}{8} \end{aligned}$$

Practise Now 12b

Practise Now 10

(a) $7\frac{1}{2} + -3\frac{3}{5} = 3\frac{9}{10}$

(b) $-2\frac{3}{4} + -\frac{5}{6} - -\frac{2}{3} = -2\frac{11}{12}$

Practise Now 11

(a) $2\frac{2}{3} \times \frac{9}{4} = 6$

(b) $4\frac{1}{6} \div \frac{5}{2} = 1\frac{2}{3}$

Practise Now 12a

(a) $5\frac{1}{4} \div -2\frac{4}{5} = -1\frac{7}{8}$

(b) $1\frac{3}{4} \times \frac{6}{5} + -\frac{1}{2} = 1\frac{9}{40}$

Practise Now 13

(a)

$$\begin{array}{r} 13.56 \\ \times \quad 2.4 \\ \hline 5\,424 \\ +27\,12\; \\ \hline 32.544 \\ \hline \end{array}$$

$\therefore 13.56 \times 2.4 = 32.544$

(b)

$$\begin{array}{r} 137.8 \\ \times \quad 0.35 \\ \hline 6\,890 \\ +41\,34\; \\ \hline 48.230 \\ \end{array}$$

$\therefore 137.8 \times 0.35 = 48.23$

Practise Now 14

(a) $0.92 \div 0.4 = \dfrac{0.92}{0.4}$

$= \dfrac{9.2}{4}$

$$\begin{array}{r} 2.3 \\ 4\overline{)\,9.2} \\ -8\; \\ \hline 1\,2 \\ -1\,2 \\ \hline 0 \end{array}$$

$\therefore 0.92 \div 0.4 = 2.3$

(b) $1.845 \div 0.15 = \dfrac{1.845}{0.15}$

$= \dfrac{184.5}{15}$

$$\begin{array}{r} 12.3 \\ 15\overline{)\,184.5} \\ -15\;\;\; \\ \hline 34\;\; \\ -30\;\; \\ \hline 4\,5 \\ -4\,5 \\ \hline 0 \\ \hline \end{array}$$

$\therefore 1.845 \div 0.15 = 12.3$

Practise Now 15

(a) $32 - (-1.6) = 32 + 1.6$

$= 33.6$

(b) $1.3 + (-3.5) = -2.2$

(c) $\dfrac{0.12}{0.4} \times \dfrac{-0.23}{0.6} = \dfrac{1.2}{4} \times \dfrac{-0.23}{0.6}$

$= 0.3 \times \dfrac{-0.23}{0.6}$

$= {}^{1}\cancel{3} \times \dfrac{-0.23}{\cancel{6}_2}$

$= -0.115$

(d) $-0.3^2 \times \dfrac{4.5}{-2.7} - 0.65 = -0.3^2 \times \dfrac{45}{-27} - 0.65$

$= {}^{0.03}\cancel{-0.09} \times \dfrac{\cancel{45}^{5}}{-\cancel{27}_{\cancel{3}\,1}} - 0.65$

$= 0.15 - 0.65$

$= -0.5$

Exercise 2A

1. **(i)** 10 001, 4 **(ii)** –12, –2017

(iii) $\dfrac{1}{5}$, 4.33, 10 001, 4 **(iv)** $-0.3, -\dfrac{5}{7}, -12, -1\dfrac{1}{2}, -2017$

2. **(a)** 30 m above sea level

(b) –350

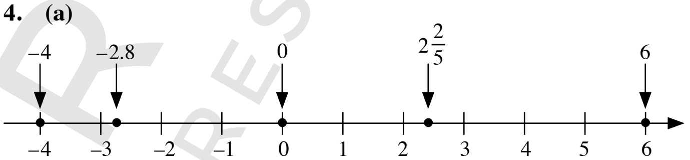
(c) An anticlockwise rotation of 30°

(d) A speed of 45 km/h of a car travelling West

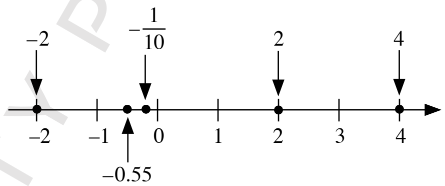
3. **(a)** < **(b)** < **(c)** <

(d) > **(e)** < **(f)** >

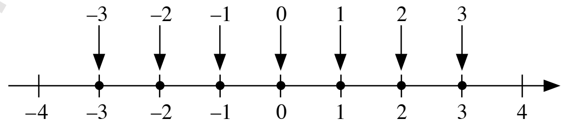
4. **(a)**



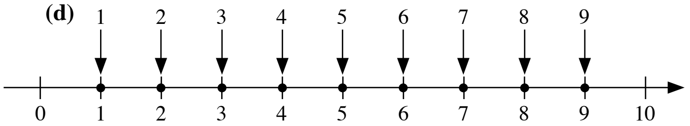
(b)



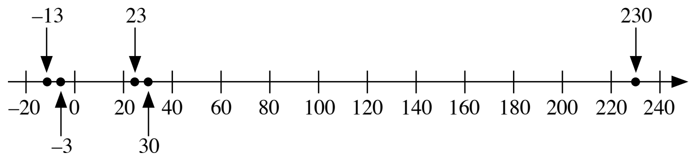
(c)



(d)

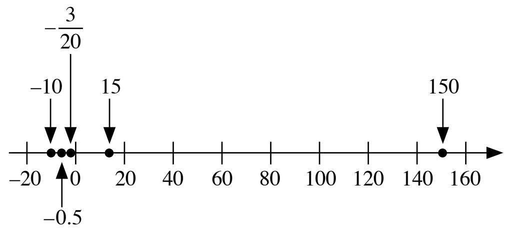


5. **(a)**



$\therefore -13, -3, 23, 30, 230$

(b)



$\therefore -10, -0.5, -\frac{3}{20}, 15, 150$

6. **(a)** 130, 32, 0, –18, –145 **(b)** 20, 3.7, –4.50, –5.6

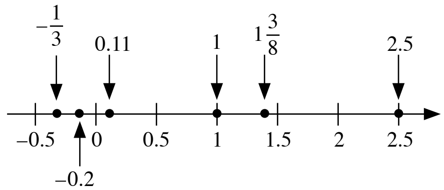
7. **(a)** –273.15 °C

(b) –86 m

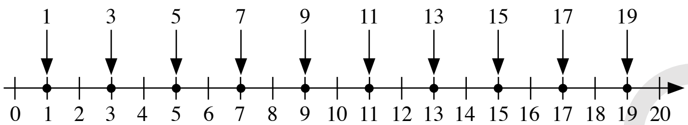
8. **(a)** > **(b)** >

(c) < **(d)** >

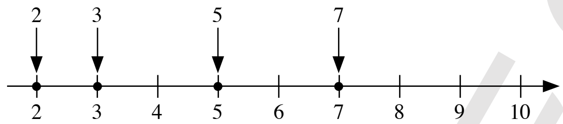
9. **(a)**



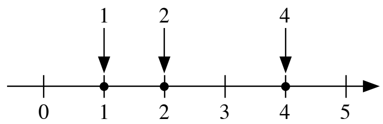
(b)



(c)



(d)



Exercise 2B

1. **(a)** 6 + (–2) = 4

(b) –5 + 8 = 3

(c) 4 + (–10) = –6

(d) –1 + (–7) = –8

(e) 9 + (–3) = 6

(f) –11 + (–5) = –16

(g) –10 + 2 = –8

(h) 1 + (–8) = –7

2. **(a)** –(–7) = 7

(b) 5 – (–3) = 5 + 3
= 8

(c) –4 – 7 = –11

(d) –8 – (–2) = –8 + 2
= –6

(e) –1 – (–10) = –1 + 10
= 9

(f) 6 – 9 = –3

(g) –8 – 3 = –11

(h) 2 – (–7) = 2 + 7
= 9

3. **(a)** 4 + (–7) – (–3) = 4 + (–7) + 3
= 0

(b) –3 – 5 + (–9) = –17

(c) 1 – 8 – (–8) = 1 – 8 + 8
= 1

(d) –2 + (–1) – 6 = –9

(e) 8 – (–9) + 1 = 8 + 9 + 1
= 18

(f) –5 + (–3) + (–2) = –10

(g) 6 + (–5) – (–8) = 6 + (–5) + 8
= 9

(h) 2 – (–7) – 8 = 2 + 7 – 8
= 1

4. **(a)** 23 + (–11) = 12

(b) –19 + 12 = –7

(c) 17 + (–29) = –12

(d) –21 + (–25) = –46

(e) –13 + 18 = 5

(f) –24 + (–13) = –37

(g) 16 + (–27) = –11

(h) –26 + 14 = –12

5. **(a)** 22 – (–13) = 22 + 13
= 35

(b) –14 – 16 = –30

(c) –19 – (–11) = –19 + 11
= –8

(d) –18 – (–22) = –18 + 22
= 4

(e) 17 – 23 = –6

(f) –20 – 15 = –35

(g) 12 – (–17) = 12 + 17
= 29

(h) –21 – 17 = –38

6. Temperature in the morning = –11 °C + 7 °C
= –4 °C

7. Point *A* shows –7 °C.
Point *B* shows 16 °C.
Difference in temperature = 16 °C – (–7 °C)
= 16 °C + 7 °C
= 23 °C

8. Altitude of town = –51 m
Difference in altitude = 138 m – (–51 m)
= 138 m + 51 m
= 189 m

9. **(i)** Difference between –2 and 3 = 3 – (–2)

$= 3 + 2$

$= 5$

(ii) The timeline for BC and AD does not have a zero while the number line has a zero.

(iii) There are 4 years between 2 BC and 3 AD.

Note: As there is no zero on the timeline, we cannot use 3 – (–2) to find the difference between 2 BC and 3 AD. In fact, the calculation should be 3 – (–2) – 1, provided one year is in BC and the other year is in AD. If both are in BC, or both are in AD, the calculation is the same as that in **(i)**.

(iv) A real-life example is the floors in a building, i.e. we can consider B1 (Basement 1) as –1 but there is no floor with the number 0.

Exercise 2C

1. **(a)** $3 \times (-19) = -57$
   (b) $-81 \times 24 = -432$
   (c) $-17 \times (-15) = 225$
   (d) $-11 \times (-16) = 76$
   (e) $-22(-7) = 154$
   (f) $-15 \times 0 = 0$
2. **(a)** $-21 \div 7 = -3$
   (b) $-144 \div 12 = -12$
   (c) $-38 \div (-19) = 2$
   (d) $\frac{-14}{2} = -7$
   (e) $\frac{15}{-5} = -3$
   (f) $\frac{-185}{-5} = 37$
3. **(a)** $-55 + (-10) - 10 = -65 - 10$

   $= -75$

   (b) $-12 - [(-8) - (-2)] + 3 = -12 - [(-8) + 2] + 3$

   $= -12 - (-6) + 3$

   $= -12 + 6 + 3$

   $= -6 + 3$

   $= -3$

   (c) $-100 + (-45) + (-5) + 20 = -145 + (-5) + 20$

   $= -150 + 20$

   $= -130$

   (d) $\frac{-2}{3} + 3 \times \frac{15}{4}$

   $= \frac{-2}{3} + \frac{45}{4}$

   $= \frac{-8}{12} + \frac{135}{12}$

   $= \frac{127}{12}$

   $= 10\frac{7}{12}$

   (e) $(\frac{-5}{8} - \frac{2}{4}) \times (\frac{3}{4})$

   $= (\frac{-5}{8} - \frac{4}{8}) \times (\frac{3}{4})$

   $= \frac{-9}{8} \times (\frac{3}{4})$

   $= \frac{-27}{32}$

   (f) $-25 \times (-4) \div (-12 + 32) = -25 \times (-4) \div 20$

   $= 100 \div 20$

   $= 5$

   (g) $3 \times (-3)^2 - (7 - 2)^2 = 3 \times (-3)^2 - 5^2$

   $= 3 \times 9 - 25$

   $= 27 - 25$

   $= 2$

   (h) $\frac{15}{4} \times [3 \times (-2) - 10\frac{1}{3}]$

   $= \frac{15}{4} \times [3 \times (-2) - \frac{31}{3}]$

   $= \frac{15}{4} \times [-6 - \frac{31}{3}]$

   $= \frac{15}{4} \times [\frac{-18}{3} - \frac{31}{3}]$

   $= \frac{15}{4} \times [\frac{-49}{3}]$

   $= \frac{\cancel{15}^{5}}{4} \times [\frac{-49}{\cancel{3}_{1}}]$

   $= \frac{-245}{4}$

   $= 61\frac{1}{4}$

   (i) $-12 \div [2^2 - (-2)] = -12 \div [4 - (-2)]$

   $= -12 \div (4 + 2)$

   $= -12 \div 6$

   $= -2$

   (j) $\sqrt{10 - 3 \times (-2)} = \sqrt{10 - (-6)}$

   $= \sqrt{10 + 6}$

   $= \sqrt{16}$

   $= 4$
4. **(a)** $-55 + (-10) - 10 = -75$
   (b) $-12 - [(-8) - (-2)] + 3 = -3$
   (c) $-100 + (-45) + (-5) + 20 = -130$
   (d) $\frac{-2}{3} + 3 \times \frac{15}{4} = 10\frac{7}{12}$
   (e) $(\frac{-5}{8} - \frac{-2}{4}) \times (\frac{3}{4}) = \frac{-27}{32}$
   (f) $-25 \times (-4) \div (-12 + 32) = 5$
   (g) $3 \times (-3)^2 - (7 - 2)^2 = 2$
   (h) $\frac{-15}{4} \times [3 \times (-2) - 10\frac{1}{3}] = -61\frac{1}{4}$
   (i) $-12 \div [2^2 - (-2)] = -2$
   (j) $\sqrt{10 - 3 \times (-2)} = 4$
5. **(a)** $24 \times (-2) \times 5 \div (-6) = -48 \times 5 \div (-6)$

   $= -240 \div (-6)$

   $= 40$

   (b) $4 \times 10 - 13 \times (-5) = 40 - (-65)$

   $= 40 + 65$

   $= 105$

   (c) $(16 - 24) - (57 - 77) \div (-2) = (-8) - (-20) \div (-2)$

   $= (-8) - 10$

   $= -18$

   (d) $160 \div (-40) - 20 \div (-5) = -4 - (-4)$

   $= -4 + 4$

   $= 0$

(e) $[(12 - 18) \div 3 - 5] \times (-4) = (-6 \div 3 - 5) \times (-4)$
$= (-2 - 5) \times (-4)$
$= (-7) \times (-4)$
$= 28$

(f) $\{[(-15 + 5) \times 2 + 8] - 32 \div 8\} - (-7)$
$= \{[(-10) \times 2 + 8] - 32 \div 8\} - (-7)$
$= [(-20 + 8) - 32 \div 8] - (-7)$
$= [(-12) - 32 \div 8] - (-7)$
$= [(-12) - 4] - (-7)$
$= (-16) - (-7)$
$= (-16) + 7$
$= -9$

(g) $(5 - 2)^3 \times 2 + [-4 + (-7)] \div (-2 + 4)^2 = 3^3 \times 2 + (-11) \div 2^2$
$= 27 \times 2 + (-11) \div 4$
$= 54 + \; -2\frac{3}{4}$
$= 51\frac{1}{4}$

(h) $\{-10 - [12 + (-3)^2] + 3^3\} \div (-3)$
$= [-10 - (12 + 9) + 3^3] \div (-3)$
$= (-10 - 21 + 3^3) \div (-3)$
$= (-10 - 21 + 27) \div (-3)$
$= (-31 + 27) \div (-3)$
$= (-4) \div (-3)$
$= 1\frac{1}{3}$

6. **(a)** $24 \times (-2) \times 5 \div (-6) = 40$
(b) $4 \times 10 - 13 \times (-5) = 105$
(c) $(16 - 24) - (57 - 77) \div (-2) = -18$
(d) $160 \div (-40) - 20 \div (-5) = 0$
(e) $[(12 - 18) \div 3 - 5] \times (-4) = 28$
(f) $\{[(-15 + 5) \times 2 + 8] - 32 \div 8\} - (-7) = -9$
(g) $(5 - 2)^3 \times 2 + [-4 + (-7)] \div (-2 + 4)^2 = 51\frac{1}{4}$
(h) $\{-10 - [12 + (-3)^2] + 3^3\} \div (-3) = 1\frac{1}{3}$

7. $\sqrt[3]{-2 \times (-6.5) - [-2 \times (-3) + 8 \times (-2) - 8 \times 2] + 5^2}$
$= \sqrt[3]{-2 \times (-6.5) - [6 + (-16) - 16] + 5^2}$
$= \sqrt[3]{-2 \times (-6.5) - (-10 - 16) + 5^2}$
$= \sqrt[3]{-2 \times (-6.5) - (-26) + 5^2}$
$= \sqrt[3]{-2 \times (-6.5) - (-26) + 25}$
$= \sqrt[3]{13 - (-26) + 25}$
$= \sqrt[3]{13 + 26 + 25}$
$= \sqrt[3]{39 + 25}$
$= \sqrt[3]{64}$
$= 4$

Exercise 2D

1. **(a)** $-\frac{1}{2} + \; -\frac{3}{4} = -\frac{1}{2} - \frac{3}{4}$
$= -\frac{2}{4} - \frac{3}{4}$
$= \frac{-2 - 3}{4}$
$= \frac{-5}{4}$
$= -1\frac{1}{4}$

(b) $3.25 + (-0.25)$
$= 3.25 - 0.25$
$= 3.00$

(c) $5\frac{1}{5} - 4\frac{1}{2} = 5\frac{2}{10} - 4\frac{5}{10}$
$= 4 + \frac{10}{10} + \frac{2}{10} - 4\frac{5}{10}$
$= \frac{7}{10}$

(d) $-3.9 + (-4.66)$
$= -3.9 - 4.66$
$= -8.56$

2. **(a)** $-\frac{1}{2} + \; -\frac{3}{4} = -1\frac{1}{4}$

(b) $3.25 + (-0.25) = 3.00$

(c) $5\frac{1}{5} - 4\frac{1}{2} = \frac{7}{10}$

(d) $-3.9 + (-4.66) = -8.56$

3. **(a)** 1.58×1.75

$$\begin{array}{r} 1\,5\,.\,8 \\ \times\, 1\,.\,7\,5 \\ \hline 7\,9\,0 \\ 1\,1\,0\,6\,0 \\ +\,1\,5\,8\,0\,0 \\ \hline 2\,7.6\,5\,0 \end{array}$$

$\therefore\ 1.58 \times 1.75 = 27.65$

(b) 2.6×0.5

$$\begin{array}{r} 2\,.\,6 \\ \times\, 0\,.\,5 \\ \hline 13.0 \\ \hline \end{array}$$

$\therefore\ 2.6 \times 0.5 = 13.0$

(c) $3.25 \div 2.5$
$= \frac{3.25}{2.5} = \frac{32.5}{25}$
$= 1.3$
$\therefore\ 3.25 \div 2.5 = 1.3$

(d) $1.73 \div 3.46$

$= \frac{1.73}{3.46} = \frac{173}{346}$

```
       0.5
346) 1 7 3
    −0 0 0
     1 7 3 0
    −1 7 3 0
           0
```

$\therefore 1.73 \div 3.46 = 0.5$

4. **(a)** $1.58 \times 1.75 = 27.65$

(b) $2.6 \times 0.5 = 13.0$

(c) $3.25 \div 2.5 = 1.3$

(d) $1.73 \div 3.46 = 0.5$

5. **(a)** $\frac{^{8}\cancel{64}}{_{5}\cancel{15}} \times -\frac{\cancel{3}^{1}}{\cancel{8}_{1}} = -\frac{8}{5}$

$= -1\frac{3}{5}$

(b) $\frac{4}{15} \div -\frac{10}{3} = \frac{^{2}\cancel{4}}{_{5}\cancel{15}} \times -\frac{\cancel{3}^{1}}{\cancel{10}_{5}}$

$= -\frac{2}{25}$

(c) $-6\frac{1}{8} \times \frac{3}{14} = -\frac{^{7}\cancel{49}}{8} \times \frac{3}{\cancel{14}_{2}}$

$= -\frac{21}{16}$

$= -1\frac{5}{16}$

(d) $-2\frac{1}{2} \times 4\frac{2}{5} = -\frac{^{1}\cancel{5}}{_{1}\cancel{2}} \times \frac{\cancel{22}^{11}}{\cancel{5}_{1}}$

$= -11$

(e) $-1\frac{1}{4} \div \frac{3}{8} = -\frac{5}{4} \div \frac{3}{8}$

$= -\frac{5}{_{1}\cancel{4}} \times \frac{\cancel{8}^{2}}{3}$

$= -\frac{10}{3}$

$= -3\frac{1}{3}$

(f) $-\frac{8}{9} \div -1\frac{2}{3} = -\frac{8}{9} \div -\frac{5}{3}$

$= -\frac{8}{_{3}\cancel{9}} \times -\frac{\cancel{3}^{1}}{5}$

$= \frac{8}{15}$

6. **(a)** $\frac{64}{15} \times -\frac{3}{8} = -1\frac{3}{5}$

(b) $\frac{4}{15} \div -\frac{10}{3} = -\frac{2}{25}$

(c) $-6\frac{1}{8} \times \frac{3}{14} = -1\frac{5}{16}$

(d) $-2\frac{1}{2} \times 4\frac{2}{5} = -11$

(e) $-1\frac{1}{4} \div \frac{3}{8} = -3\frac{1}{3}$

(f) $-\frac{8}{9} \div -1\frac{2}{3} = \frac{8}{15}$

7. **(a)**

```
   14 .72
×     1.2
   2 944
+14 72
  17.664
```

$\therefore 14.72 \times 1.2 = 17.664$

(b)

```
  1 3 0 . 4
×   0 . 1 5
    6 5 2 0
+ 1 3 0 4
  1 9 . 5 6 0
```

$\therefore 130.4 \times 0.15 = 19.56$

(c)

```
   0 . 2 7
×  0 . 0 8
 0.0 2 1 6
```

$\therefore 0.27 \times 0.08 = 0.0216$

(d)

```
    0 . 25
×   1 . 96
       150
      225
+    25
   0 . 4900
```

$\therefore 0.25 \times 1.96 = 0.49$

8. **(a)** $0.81 \div 0.3 = \frac{0.81}{0.3}$

$= \frac{8.1}{3}$

```
    2.7
3 ) 8.1
   −6
    2 1
   −2 1
      0
```

$\therefore 0.81 \div 0.3 = 2.7$

(b) $1.32 \div 0.12 = \frac{1.32}{0.12}$

$= \frac{132}{12}$

$= 11$

(c) $3.426 \div 0.06 = \dfrac{3.426}{0.06}$

$= \dfrac{342.6}{6}$

$$\begin{array}{r} 57.1 \\ 6\overline{)\,342.6} \\ -30 \\ \hline 42 \\ -42 \\ \hline 6 \\ -6 \\ \hline 0 \end{array}$$

$\therefore 3.426 \div 0.06 = 57.1$

(d) $4.35 \div 1.5 = \dfrac{4.35}{1.5}$

$= \dfrac{43.5}{15}$

$$\begin{array}{r} 2.9 \\ 15\overline{)\,43.5} \\ -30 \\ \hline 135 \\ -135 \\ \hline 0 \end{array}$$

$\therefore 4.35 \div 1.5 = 2.9$

9. **(a)** $4.3 - (-3.9) = 4.3 + 3.9$
$= 8.2$

(b) $2.8 + (-1.5) = 1.3$

(c) $-5.9 + 2.7 = -3.2$

(d) $-6.7 - 5.4 = -12.1$

Review Exercise 2

1. **(a)** $-7 - 38 = -45$
$8 + (-55) = -47$
$\therefore -7 - 38 > 8 + (-55)$

(b) $2.36 - 10.58 = -8.22$
$-11.97 - (-2.69) = -11.97 + 2.69$
$= -9.28$
$\therefore 2.36 - 10.58 > -11.97 - (-2.69)$

(c) $-5 \times 1.5 = -7.5$
$50 \div (-8) = -6.25$
$\therefore -5 \times 1.5 < 50 \div (-8)$

2. **(a)** $13 - (-54) = 13 + 54$
$= 67$

(b) $(-74) - (-46) = -74 + 46$
$= -28$

(c) $11 + (-33) - (-7) = -22 - (-7)$
$= -22 + 7$
$= -15$

(d) $-13 + (-15) + (-8) = -28 + (-8)$
$= -36$

3. **(a)** $-12 \times 7 = -84$

(b) $4 \times (-5) \times (-6) = -20 \times (-6)$
$= 120$

(c) $-600 \div 15 = -40$

(d) $50 \div (-8) \div (-5) = -\dfrac{50}{8} \div (-5)$

$= -\dfrac{25}{4} \div (-5)$

$= -\dfrac{\cancel{25}^{5}}{4} \times -\dfrac{1}{\cancel{5}_{1}}$

$= \dfrac{5}{4}$

$= 1\dfrac{1}{4}$

4. **(a)** $(-3 - 5) \times (-3 - 4) = (-8) \times (-7)$
$= 56$

(b) $4 \times (-5) \div (-2) = -20 \div (-2)$
$= 10$

(c) $-5 \times 6 - 18 \div (-3) = -30 - (-6)$
$= -30 + 6$
$= -24$

(d) $2 \times (-3)^2 - 3 \times 4 = 2 \times 9 - 3 \times 4$
$= 18 - 12$
$= 6$

(e) $-3 \times (-2) \times (2 - 5)^2 = -3 \times (-2) \times (-3)^2$
$= -3 \times (-2) \times 9$
$= 6 \times 9$
$= 54$

(f) $(-2)^2 - (-2) \times 3 + 2 \times 3^2 = 4 - (-2) \times 3 + 2 \times 9$
$= 4 - (-6) + 18$
$= 4 + 6 + 18$
$= 10 + 18$
$= 28$

(g) $(-4)^2 \div (-8) + 3 \times (-2)^3 = 16 \div (-8) + 3 \times (-8)$
$= (-2) + (-24)$
$= -26$

(h) $4 \times 3^2 \div (-6) - (-1)^3 \times (-3)^2 = 4 \times 9 \div (-6) - (-1) \times 9$
$= 36 \div (-6) - (-1) \times 9$
$= -6 - (-9)$
$= -6 + 9$
$= 3$

(i) $-2 \times (-2)^3 \times (-2) \times 3 + (-2) \times 3 \times (-1)^2$
$= -2 \times (-8) \times (-2) \times 3 + (-2) \times 3 \times 1$
$= 16 \times (-2) \times 3 + (-6)$
$= -32 \times 3 + (-6)$
$= -96 + (-6)$
$= -102$

(j) $5 - \{12 \times [(-5)^2 - 7] \div 3\} = 5 - [12 \times (25 - 7) \div 3]$
$= 5 - (12 \times 18 \div 3)$
$= 5 - (216 \div 3)$
$= 5 - 72$
$= -67$

Challenge Yourself

1. **(a)**

$$\begin{array}{r} 3\,2\,4 \\ \times \quad 5\,7 \\ \hline 2\,2\,6\,8 \\ +1\,6\,2\,0 \\ \hline 1\,8\,4\,6\,8 \\ \hline \end{array}$$

(b)

$$\begin{array}{r} 4\,8 \\ 28\overline{)\,1\,3\,4\,4} \\ -1\,1\,2 \\ \hline 2\,2\,4 \\ -2\,2\,4 \\ \hline 0 \end{array}$$

2. **(b)** $(3 + 3) \div 3 + 3 - 3 = 2$ **(c)** $3 + 3 - 3 - 3 + 3 = 3$

(d) $(3 + 3 + 3 + 3) \div 3 = 4$ **(e)** $3 + 3 \div 3 + 3 \div 3 = 5$

(f) $3 + 3 + (3 - 3) \times 3 = 6$

Chapter 3 Percentages

TEACHING NOTES

Suggested Approach

Although students have learnt percentage in primary school (i.e. how to express a part of a whole as a percentage, write fractions and decimals as percentages, and vice versa, find a percentage part of a whole and solve up to 2-step word problems involving percentage), many may still struggle with percentage. Teachers can introduce percentage as fractions by going right back to the fundamentals. Teachers can give students practical applications of percentages and show the changes in fractions and proportions through the examples to give them a better understanding of the concept.

Section 3.1: Introduction to Percentage

Teachers can get students to work in pairs to find an advertisement/article in which percentage(s) can be found and discuss about it together (see Class Discussion: Percentages in Real Life). After the discussion, students should understand the meaning of percentage(s) better and interpret information more accurately. Students need to be able to comment critically on the usefulness of percentages before they can have a confident grasp of the topic.

Teachers can then build upon what students have learnt about percentage in primary school. Students may be able to learn how to accurately calculate a percentage but they might struggle to explain the meaning behind it. Teachers should emphasise on the basics of fractions and proportions before getting the students to calculate and interpret percentages.

In Worked Example 5, students should learn that it is easy to see that more people passed the entrance test in 2011 but it is not easy to see which year had a higher proportion of people passing the entrance test. Teachers can highlight to the students that two quantities can be easily compared using percentages because the proportions are converted to the same base i.e. 100.

Section 3.2: Percentage Change and Reverse Percentage

Teachers should guide students on how to use algebra in percentage change and reverse percentage. Students may draw models, wherever applicable, to help them understand the problem.

Through the worked examples in this section, students should be able to tackle percentage change and reverse percentage problems involving algebra. They should also learn how to identify whether the problem is a reverse percentage or a percentage change problem. Teachers can highlight to the students that percentage change is when they are given both the new value and the original value while a reverse percentage is when they need to find the original value given a quantity after a percentage increase or decrease.

WORKED SOLUTIONS

Class Discussion (Percentage in Real Life)

1. **Guiding Questions:**
   - What is the advertisement/article about?
   - Are the percentages found in the advertisement/article expressed using the percentage symbol or in words?
   - What do the percentages mean in the context of the advertisement/article?

   Teachers may use this to assess students' prior knowledge of percentage, e.g. whether students are able to relate percentages to fractions and to perform relevant calculations using the given percentages to illustrate the meaning of the percentages in the context of the advertisement/article. Teachers may also use this as a trigger to show students the need to learn percentage, and link back to the different scenarios in the advertisement/article.
   Alternatively, teachers may wish to use the article titled 'A smaller and cheaper iPad' (Today, 5 July 2012) and/or the apparel advertisement (Page 3, Today, 5 July 2012) for this question.

   The article 'A smaller and cheaper iPad?' is about Apple's apparent intention to launch an iPad which is smaller and less expensive. This is in order to counter its rivals' new products so as to maintain its stronghold in the tablet market. The percentage found in the article is 61 per cent, which is expressed in words and in the context of the article, it means that Apple has 61% of the tablet market share. A conclusion that can be drawn is that the iPad is the most popular tablet in the market as the total tablet market share is the base, i.e. 100%. The other brands have a total of 100% – 61% = 39% of the tablet market share.

   The advertisement shows that a particular brand of apparel is holding a storewide end-of-season sale. The percentages found in the advertisement are 70% and 10%, which are expressed using the percentage symbol. In the context of the advertisement, it means that shoppers can enjoy a discount of up to 70% on the items and those with credit cards are entitled to an additional 10% off if they purchase a minimum of 3 items.

2. **Guiding Questions:**
   - What is the meaning of the term 'up to'?
   - An advertisement with the phrase 'Discount up to 80% on All Items' is displayed at the entrance of a shop. If an item in the shop is sold at a discount of 10%, does this mean that the shopkeeper is dishonest?
   - Is it true that all the items in the shop are sold at a discount? Could there be any exceptions?
   - Does it mean that the prices of all the items in the shop are very low?

   The term 'up to' in the phrase 'Discount up to 80% on All Items' suggests that the greatest percentage discount given on the items in the shop is 80%. This means that some items in the shop may be sold at a discount that is less than 80%. Hence, the shopkeeper is not dishonest.
   Most of the time, such advertisements come with terms and conditions that may state the items which are not subjected to a discount, such as 'New Arrivals'. These terms and conditions are normally shown in fine print.
   As some items may be sold at a discount of less than 80% and some items may not be subjected to a discount, the prices of the items in the shop may not be low. In addition, the original prices of some items may be very high, such that their prices are still high even after a discount.

   Teachers may wish to ask students to list other instances where such phrases are used. They may also want to take this opportunity to highlight to students the importance of being informed consumers. Students should not take information at face value. Instead, they should learn how to interpret information accurately.

3. **Guiding Questions:**
   The following shows examples of statements with percentages more than 100%:
   - In Singapore, the number of employers hiring ex-offenders has increased by more than 100% from 2004 to 2011.
   - The total number of registrants for a school during Phase 2B of the Primary 1 registration is 120% of the number of vacancies available.
   - The number of mobile subscriptions in Singapore in 2011 is about 150% of her population.
   - The population of Singapore in 2010 is about 240% of that in 1970.
   - The total fertility rate in Singapore in 1970 is about 270% of that in 2010.

   The phrase 'this year's sales is 200% of last year's sales' means that the sales this year is 2 times of that of last year.

   Teachers may ask students to refer to page 197 of the textbook for an example of how this phrase may be used. Teachers should also highlight to students that the use of percentages can be misleading, e.g. a salesman who sold a car in January and two cars in February can say that his sales in February is 200% of that in January.

Class Discussion (Expressing Two Quantities in Equivalent Forms)

1. **(a) (i)** Required percentage $= \frac{40}{50} \times 100\%$
   $= 80\%$
   - There are 80% as many male teachers as female teachers.
   - The number of male teachers is 80% of the number of female teachers.
   - The number of male teachers is $\frac{4}{5}$ of the number of female teachers.

   (ii) Required percentage $= \frac{50}{40} \times 100\%$
   $= 125\%$
   - There are 125% as many female teachers as male teachers.

- The number of female teachers is 125% of the number of male teachers.
- The number of female teachers is $\frac{5}{4}$ of the number of male teachers.

(b)

In words	A is 80% of B.	B is 125% of A.
Percentage	$A = 80\% \times B$	$B = 125\% \times A$
Fraction	$A = \frac{4}{5}$ (fraction) $\times B$	$B = \frac{5}{4}$ (fraction) $\times A$
Decimal	$A = 0.8 \times B$	$B = 1.25 \times A$

Table 3.1

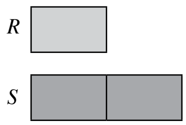
2. (i)

In words	P is 20% of Q.	R is 50% of S.	T is 125% of U.
Percentage	$P = 20\% \times Q$	$R = 50\% \times S$	$T = 125\% \times U$
Fraction	$P = \frac{1}{5}$ (fraction) $\times Q$	$R = \frac{1}{2}$ (fraction) $\times S$	$T = \frac{5}{4}$ (fraction) $\times U$
Decimal	$P = 0.2 \times Q$	$R = 0.5 \times S$	$T = 1.25 \times Q$

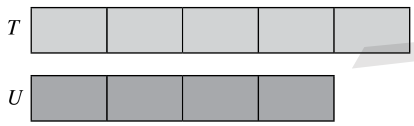
Table 3.2

(ii) The relationship between P and Q can be illustrated as follows:

The relationship between R and S can be illustrated as follows:



The relationship between T and U can be illustrated as follows:



Thinking Time (Page 61)

1. No, it is not correct to say that $\frac{20\% + 80\%}{2}$, i.e. 50% of the total number of students in the two groups had done the survey. This is because there may be a different number of students in each of the two groups, e.g. if Ahsan conducted the survey on 20% of a group of 100 students and on 80% of another group of 200 students, then $\frac{20\% \times 100 + 80\% \times 200}{100 + 200} \times 100\% = 60\%$ of the total number of students in the two groups had done the survey.

2. Mr Faiz's monthly salary in 2011 $= 110\% \times \text{PKR } x$

$$= \frac{110}{100} \times \text{PKR } x$$
$$= \text{PKR } 1.1x$$

Mr Faiz's monthly salary in 2012 $= 90\% \times \text{PKR } 1.1x$

$$= \frac{90}{100} \times \text{PKR } 1.1x$$
$$= \text{PKR } 0.99x$$

Hence, it is not correct to say that Mr Faiz's monthly salary in 2012 was PKR x.

Practise Now 1

(a) (i) $45\% = \frac{45}{100}$

$= \frac{9}{20}$

(ii) $305\% = \frac{305}{100}$

$= \frac{61}{20}$

$= 3\frac{1}{20}$

(iii) $5.5\% = \frac{5.5}{100}$

$= \frac{55}{1000}$

$= \frac{11}{200}$

(iv) $8\frac{5}{7}\% = \frac{61}{7}\%$

$= \frac{61}{7} \div 100$

$= \frac{61}{7} \times \frac{1}{100}$

$= \frac{61}{700}$

(b) (i) $\frac{17}{20} = \frac{17}{20} \times 100\%$

$= \frac{1700}{20}$

$= 85\%$

(ii) $23\frac{1}{5} = \frac{116}{5}$

$= \frac{116}{5} \times 100\%$

$= \frac{11\,600}{5}\%$

$= 2320\%$

Practise Now 2

(a) (i) $12\% = \frac{12}{100}$

$= 0.12$

(ii) $413\% = \frac{413}{100}$

$= 4.13$

(iii) $23.6\% = \dfrac{23.6}{100}$
$= 0.236$

(iv) $6\dfrac{1}{4}\% = \dfrac{25}{4}\%$
$= \dfrac{25}{4} \div 100$
$= \dfrac{25}{4} \times \dfrac{1}{100}$
$= \dfrac{25}{400}$
$= 0.0625$

Alternatively,
$6\dfrac{1}{4}\% = 6.25\%$
$= \dfrac{6.25}{100}$
$= 0.0625$

(b) **(i)** $0.76 = 0.76 \times 100\%$
$= 76\%$

(ii) $2.789 = 2.789 \times 100\%$
$= 278.9\%$

Practise Now 3

1. **(i)** Total number of teachers in the school $= 45 + 75$
$= 120$

Percentage of male teachers in the school $= \dfrac{45}{120} \times 100\%$
$= 37.5\%$

(ii) **Method 1:**

Percentage of female teachers in the school $= \dfrac{75}{120} \times 100\%$
$= 62.5\%$

Method 2:

Percentage of female teachers in the school $= 100\% - 37.5\%$
$= 62.5\%$

2. Required percentage $= \dfrac{1400\ \text{m}l}{2.1\ l} \times 100\%$
$= \dfrac{1400\ \text{m}l}{2100\ \text{m}l} \times 100\%$
$= \dfrac{2}{3} \times 100\%$
$= 66\dfrac{2}{3}\%$

Practise Now 4

1. **(a)** 20% of PKR 13.25 $= 20\% \times$ PKR 13.25
$= \dfrac{20}{100} \times$ PKR 13.25
$=$ PKR 2.65

(b) $15\dfrac{3}{4}\%$ of PKR 640 $= 15\dfrac{3}{4}\% \times$ PKR 640
$= \dfrac{15\frac{3}{4}}{100} \times$ PKR 640
$=$ PKR 100.80

2. 2500% of PKR 460 $= 2500\% \times$ PKR 460
$= \dfrac{2500}{100} \times$ PKR 460
$=$ PKR 11 500

Practise Now 5

1. **Method 1:**

Number of students who were late for school $= 3\% \times 1500$
$= \dfrac{3}{100} \times 1500$
$= 45$

Number of students who were punctual for school $= 1500 - 45$
$= 1455$

Method 2:

Percentage of students who were punctual for school $= 100\% - 3\%$
$= 97\%$

Number of students who were punctual for school $= 97\% \times 1500$
$= \dfrac{97}{100} \times 1500$
$= 1455$

2. Percentage of children who attended the dinner
$= 100\% - 35.5\% - 40\%$
$= 24.5\%$

Number of children who attended the dinner $= 24.5\% \times 1800$
$= \dfrac{24.5}{100} \times 1800$
$= 441$

Practise Now 6

Percentage of people who attended the New Year party in Village *A*
$= \dfrac{4000}{30\,000} \times 100\%$
$= 13\dfrac{1}{3}\%$

Percentage of people who attended the New Year party in Village *B*
$= \dfrac{2800}{25\,000} \times 100\%$
$= 11.2\%$

$\therefore$ Village A had a higher percentage of people who attended its New Year party.

Practise Now 7

1. **(a)** Value of award for a Secondary 1 student in 2009
$= 140\% \times$ PKR 2500
$= \dfrac{140}{100} \times$ PKR 2500
$=$ PKR 3500

(b) (i) Percentage increase in value of award from 2008 to 2009 for a Primary 1 student

$= \dfrac{\text{PKR } 2500 - \text{PKR } 1500}{\text{PKR } 1500} \times 100\%$

$= \dfrac{\text{PKR } 1000}{\text{PKR } 1500} \times 100\%$

$= 66\dfrac{2}{3}\%$

(ii) Percentage increase in value of award from 2008 to 2009 for a Primary 6 student

$= \dfrac{\text{PKR } 3000 - \text{PKR } 2000}{\text{PKR } 2000} \times 100\%$

$= \dfrac{\text{PKR } 1000}{\text{PKR } 2000} \times 100\%$

$= 50\%$

2. (a) Required result $= 75\% \times 32$

$= \dfrac{75}{100} \times 32$

$= 24$

(b) Percentage decrease in value of motorcycle

$= \dfrac{\text{PKR } 127\,000 - \text{PKR } 119\,380}{\text{PKR } 127\,000} \times 100\%$

$= \dfrac{\text{PKR } 7620}{\text{PKR } 127\,000} \times 100\%$

$= 6\%$

Practise Now 8

	Original Cost	Percentage Change	New Cost
Rental	PKR 2400	–5%	$\frac{95}{100} \times$ PKR 2400 = PKR 2280
Wages	PKR 1800	–6%	$\frac{94}{100} \times$ PKR 1800 = PKR 1692
Utilities	PKR 480	+7%	$\frac{107}{100} \times$ PKR 480 = PKR 513.60
Business	PKR 4680		PKR 4485.60

Percentage decrease in monthly cost of running business

$= \dfrac{\text{PKR } 4680 - \text{PKR } 4485.60}{\text{PKR } 4680} \times 100\%$

$= \dfrac{\text{PKR } 194.40}{\text{PKR } 4680} \times 100\%$

$= 4\dfrac{2}{13}\%$

Practise Now 9

70% of the books $= 35$

1% of the books $= \dfrac{35}{70}$

100% of the books $= \dfrac{35}{70} \times 100$

$= 50$

There are 50 books on the bookshelf.

Practise Now 10

1. Method 1:

109% of original cost = PKR 654

1% of original cost $= \dfrac{\text{PKR } 654}{109}$

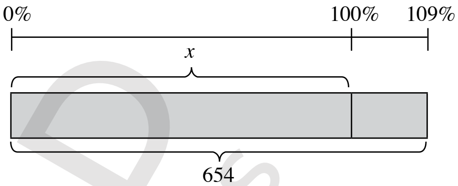
100% of original cost $= \dfrac{\text{PKR } 654}{109} \times 100$

= PKR 600

The original cost of the article is PKR 600.

Method 2:

Let the original cost of the article be PKR x.



From the model, we form the equation:

$109\% \times x = 654$

$1.09x = 654$

$x = 600$

The original cost of the article is PKR 600.

2. 120% of value in 2011 = PKR 180 000

1% of value in 2011 $= \dfrac{\text{PKR } 180\,000}{120}$

100% of value in 2011 $= \dfrac{\text{PKR } 180\,000}{120} \times 100$

= PKR 150 000

The value of the vase was PKR 150 000 in 2011.

120% of value in 2010 = PKR 150 000

1% of value in 2010 $= \dfrac{\text{PKR } 150\,000}{120}$

100% of value in 2010 $= \dfrac{\text{PKR } 150\,000}{120} \times 100$

= PKR 125 000

The value of the vase was PKR 125 000 in 2010.

Practise Now 11

1. Method 1:

97% of original monthly salary = PKR 3346.50

1% of original monthly salary $= \dfrac{\text{PKR } 3346.50}{97}$

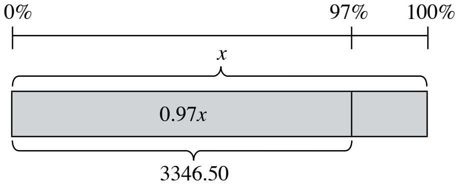
100% of original monthly salary $= \dfrac{\text{PKR } 3346.50}{97} \times 100$

= PKR 3450

Bina's original monthly salary is PKR 3450.

Method 2:

Let Bina's original monthly salary be PKR x.



From the model, we form the equation:

$$97\% \times x = 3346.50$$
$$0.97x = 3346.50$$
$$x = 3450$$

Bina's original monthly salary is PKR 3450.

2. 85% of value in 2011 $=$ PKR 86 700

$$1\% \text{ of value in 2011} = \frac{\text{PKR } 86\,700}{85}$$

$$100\% \text{ of value in 2011} = \frac{\text{PKR } 86\,700}{85} \times 100$$
$$= \text{PKR } 102\,000$$

The value of the car was PKR 102 000 in 2011.

85% of value in 2010 $=$ PKR 102 000

$$1\% \text{ of value in 2010} = \frac{\text{PKR } 102\,000}{85}$$

$$100\% \text{ of value in 2010} = \frac{\text{PKR } 102\,000}{85} \times 100$$
$$= \text{PKR } 120\,000$$

The value of the car was PKR 120 000 in 2010.

Exercise 3A

1. **(a)** $28\% = \frac{28}{100} = \frac{7}{25}$

(b) $158\% = \frac{158}{100} = \frac{79}{50} = 1\frac{29}{50}$

(c) $12.4\% = \frac{12.4}{100} = \frac{124}{1000} = \frac{31}{250}$

(d) $6\frac{3}{5}\% = \frac{33}{5}\% = \frac{33}{5} \div 100 = \frac{33}{5} \times \frac{1}{100} = \frac{33}{500}$

2. **(a)** $4\% = \frac{4}{100} = 0.04$

(b) $633\% = \frac{633}{100} = 6.33$

(c) $0.02\% = \frac{0.02}{100} = 0.0002$

(d) $33\frac{2}{3}\% = \frac{101}{3}\% = \frac{101}{3} \div 100 = \frac{101}{3} \times \frac{1}{100} = \frac{101}{300} = 0.337$ (to 3 s.f.)

3. **(a)** $\frac{3}{5} = \frac{3}{5} \times 100\% = 60\%$

(b) $\frac{9}{10} = \frac{9}{10} \times 100\% = 90\%$

(c) $\frac{6}{125} = \frac{6}{125} \times 100\% = 4.8\%$

(d) $\frac{6}{5} = \frac{6}{5} \times 100\% = 120\%$

(e) $\frac{12}{25} = \frac{12}{25} \times 100\% = 48\%$

(f) $1\frac{6}{25} = \frac{31}{25} \times 100\% = 124\%$

4. **(a)** $0.78 = 0.78 \times 100\% = 78\%$

(b) $0.25 = 0.25 \times 100\% = 25\%$

(c) $0.07 = 0.07 \times 100\% = 7\%$

(d) $0.095 = 0.095 \times 100\% = 9.5\%$

(e) $1.35 = 1.35 \times 100\% = 135\%$

(f) $2.6 = 2.6 \times 100\% = 260\%$

5. **(a)** 50% of PKR 70 $= 50\% \times \text{PKR } 70 = \frac{50}{100} \times \text{PKR } 70 = \text{PKR } 35$

(b) 80% of 4.5 m = 80% × 4.5 m

$= \frac{80}{100} \times 4.5$ m

= 3.6 m

6. **(i)** Total number of students in the class = 20 + 18

= 38

Percentage of boys in the class $= \frac{20}{38} \times 100\%$

$= 52\frac{12}{19}\ \%$

(ii) Percentage of girls in the class $= 100\% - 52\frac{12}{19}\ \%$

$= 47\frac{7}{19}\ \%$

7. Percentage of cars which are not blue = 100% – 30%

= 70%

Number of cars which are not blue = 70% × 120

$= \frac{70}{100} \times 120$

= 84

8. Percentage of annual income Salman donated to charitable organisations

$= \frac{\text{PKR } 1200}{12 \times \text{PKR } 1600} \times 100\%$

$= \frac{\text{PKR } 1200}{\text{PKR } 19\,200} \times 100\%$

= 6.25%

Percentage of annual income Faiza donated to charitable organisations

$= \frac{\text{PKR } 4500}{12 \times \text{PKR } 6800} \times 100\%$

$= \frac{\text{PKR } 4500}{\text{PKR } 81\,600} \times 100\%$

$= 5\frac{35}{68}\ \%$

∴ Salman donated a higher percentage of his annual income to charitable organisations.

9. **(a)** Required percentage $= \frac{25 \text{ seconds}}{3.5 \text{ minutes}} \times 100\%$

$= \frac{25 \text{ seconds}}{210 \text{ seconds}} \times 100\%$

$= \frac{5}{42} \times 100\%$

$= 11\frac{19}{21}\ \%$

(b) Required percentage $= \frac{45 \text{ minutes}}{1 \text{ hour}} \times 100\%$

$= \frac{45 \text{ minutes}}{60 \text{ minutes}} \times 100\%$

$= \frac{3}{4} \times 100\%$

= 75%

(c) Required percentage $= \frac{1 \text{ year}}{4 \text{ months}} \times 100\%$

$= \frac{12 \text{ months}}{4 \text{ months}} \times 100\%$

= 3 × 100%

= 300%

(d) Required percentage $= \frac{15 \text{ mm}}{1 \text{ m}} \times 100\%$

$= \frac{15 \text{ mm}}{1000 \text{ mm}} \times 100\%$

$= \frac{3}{200} \times 100\%$

= 1.5%

(e) Required percentage $= \frac{335 \text{ cm}}{5 \text{ m}} \times 100\%$

$= \frac{335 \text{ cm}}{500 \text{ cm}} \times 100\%$

$= \frac{67}{100} \times 100\%$

= 67%

(f) Required percentage $= \frac{1 \text{ kg}}{800 \text{ g}} \times 100\%$

$= \frac{1000 \text{ g}}{800 \text{ g}} \times 100\%$

$= \frac{5}{4} \times 100\%$

= 125%

(g) Required percentage $= \frac{60\Upsilon}{360\Upsilon} \times 100\%$

$= \frac{1}{6} \times 100\%$

$= 16\frac{2}{3}\ \%$

(h) Required percentage $= \frac{63 \text{ paisas}}{\text{PKR } 2.10} \times 100\%$

$= \frac{63 \text{ paisas}}{210 \text{ paisas}} \times 100\%$

$= \frac{3}{10} \times 100\%$

= 30%

10. **(a)** $6\frac{1}{5}\ \%$ of 1.35 ml $= 6\frac{1}{5}\ \% \times 1.35$ ml

$= \frac{6\frac{1}{5}}{100} \times 1.35$ ml

$= \frac{837}{10\,000}$ ml

(b) $56\frac{7}{8}\ \%$ of 810 m $= 56\frac{7}{8}\ \% \times 810$ m

$= \frac{56\frac{7}{8}}{100} \times 810$ m

$= 460\frac{11}{16}$ m

(c) 0.56% of 15 000 l = 0.56% × 15 000 l

$= \frac{0.56}{100} \times 15\,000\ l$

= 84 l

(d) 2000% of $5¢ = 2000\% \times 5¢$

$= \frac{2000}{100} \times 5¢$

$= 100¢$

= PKR 1

11. Percentage of marks Kiran obtains $= \frac{40}{60} \times 100\%$

$= 66\frac{2}{3}\%$

∴ Kiran gets a bronze award.

Percentage of marks Seema obtains $= \frac{46}{60} \times 100\%$

$= 76\frac{2}{3}\%$

∴ Seema gets a silver award.

Percentage of marks Nadia obtains $= \frac{49}{60} \times 100\%$

$= 81\frac{2}{3}\%$

∴ Nadia gets a gold award.

12. Percentage of employees who were unaffected by the financial crisis

$= 100\% - 2.5\% - 50.75\%$

$= 46.75\%$

Number of employees who were unaffected by the financial crisis

$= 46.75\% \times 12\,000$

$= \frac{46.75}{100} \times 12\,000$

$= 5610$

13. Amount Ahsan spent on room rental $= 20.5\% \times$ PKR 18 500

$= \frac{20.5}{100} \times$ PKR 18 500

= PKR 3792.5

Amount Ahsan overspent = PKR 3792.5 + PKR 6900 + PKR 9400 – PKR 18 500

= PKR 1592.5

Required percentage $= \frac{\text{PKR } 1592.5}{\text{PKR } 1850} \times 100\%$

$= 8.61\%$ (to 2 d.p.)

14. Number of remaining pages after Friday $= 600 - 150$

$= 450$

Number of pages that remains to be read $= (100\% - 40\%) \times 450$

$= 60\% \times 450$

$= \frac{60}{100} \times 450$

$= 270$

Required percentage $= \frac{270}{600} \times 100\%$

$= 45\%$

Exercise 3B

1. (a) Required value $= 135\% \times 60$

$= \frac{135}{100} \times 60$

$= 81$

(b) Required value $= 225\% \times 28$

$= \frac{225}{100} \times 28$

$= 63$

(c) Required value $= 55\% \times 120$

$= \frac{55}{100} \times 120$

$= 66$

(d) Required value $= 62\frac{1}{2}\% \times 216$

$= \frac{62\frac{1}{2}}{100} \times 216$

$= 135$

2. (a) 20% of number $= 17$

1% of number $= \frac{17}{20}$

100% of number $= \frac{17}{20} \times 100$

$= 85$

The number is 85.

(b) 175% of number $= 49$

1% of number $= \frac{49}{175}$

100% of number $= \frac{49}{175} \times 100$

$= 28$

The number is 28.

(c) 115% of number $= 161$

1% of number $= \frac{161}{115}$

100% of number $= \frac{161}{115} \times 100$

$= 140$

The number is 140.

(d) 80% of number $= 192$

1% of number $= \frac{192}{80}$

100% of number $= \frac{192}{80} \times 100$

$= 240$

The number is 240.

3. Percentage increase in length of elastic band $= \frac{90 - 72}{72} \times 100\%$

$= \frac{18}{72} \times 100\%$

$= 25\%$

4. **(i)** Value of award for a Secondary 1 student in the top 5% in 2009

$= 130\% \times \text{PKR } 5000$

$= \frac{130}{100} \times \text{PKR } 5000$

$= \text{PKR } 6500$

(ii) Percentage increase in value of award from 2008 to 2009 for a Primary 4 student in the next 5%

$= \frac{\text{PKR } 3500 - \text{PKR } 2500}{\text{PKR } 2500} \times 100\%$

$= \frac{\text{PKR } 1000}{\text{PKR } 2500} \times 100\%$

$= 40\%$

5. Percentage decrease in price of meat

$= \frac{\text{PKR } 1360 - \text{PKR } 1020}{\text{PKR } 1360} \times 100\%$

$= \frac{\text{PKR } 340}{\text{PKR } 1360} \times 100\%$

$= 25\%$

6. Value of car at the end of 2010 $= 80\% \times \text{PKR } 120\ 000$

$= \frac{80}{100} \times \text{PKR } 120\ 000$

$= \text{PKR } 96\ 000$

Value of car at the end of 2011 $= 90\% \times \text{PKR } 96\ 000$

$= \frac{90}{100} \times \text{PKR } 96\ 000$

$= \text{PKR } 86\ 400$

7. 45% of the students $= 135$

1% of the students $= \frac{135}{45}$

100% of the students $= \frac{135}{45} \times 100$

$= 300$

There are 300 students who take part in the competition.

8. 136% of original cost $= \text{PKR } 333\ 200$

1% of original cost $= \frac{\text{PKR } 333\ 200}{136}$

100% of original cost $= \frac{\text{PKR } 333\ 200}{136} \times 100$

$= \text{PKR } 245\ 000$

The cost of the house when it was built is PKR 245 000.

9. 90% of original bill $= \text{PKR } 58.50$

1% of original bill $= \frac{\text{PKR } 58.50}{90}$

100% of original bill $= \frac{\text{PKR } 58.50}{90} \times 100$

$= \text{PKR } 65$

The original bill is PKR 65.

10. Value obtained after initial increase $= 130\% \times 2400$

$= \frac{130}{100} \times 2400$

$= 3120$

Final number $= 80\% \times 3120$

$= \frac{80}{100} \times 3120$

$= 2496$

11. Let the number of train passengers in 2010 be x.

Number of train passengers in 2011 $= 108\% \times x$

$= \frac{108}{100} \times x$

$= 1.08x$

Number of train passengers in 2012 $= 108\% \times 1.08x$

$= \frac{108}{100} \times 1.08x$

$= 1.1664x$

Percentage increase in number of train passengers from 2010 to 2012

$= \frac{1.1664x - x}{x} \times 100\%$

$= \frac{0.1664x}{x} \times 100\%$

$= 0.1664 \times 100\%$

$= 16.64\%$

12.

	Original Cost	Percentage Change	New Cost
Raw materials	PKR 100	+11%	$\frac{111}{100} \times \text{PKR } 100 = \text{PKR } 111$
Overheads	PKR 80	+20%	$\frac{120}{100} \times \text{PKR } 80 = \text{PKR } 96$
Wages	PKR 120	–15%	$\frac{85}{100} \times \text{PKR } 120 = \text{PKR } 102$
Printer	PKR 300		PKR 309

Percentage increase in production cost of printer

$= \frac{\text{PKR } 309 - \text{PKR } 300}{\text{PKR } 300} \times 100\%$

$= \frac{\text{PKR } 9}{\text{PKR } 300} \times 100\%$

$= 3\%$

13. 115% of value in 2011 $= \text{PKR } 899\ 300$

1% of value in 2011 $= \frac{\text{PKR } 899\ 300}{115}$

100% of value in 2011 $= \frac{\text{PKR } 899\ 300}{115} \times 100$

$= \text{PKR } 782\ 000$

The value of the condominium was PKR 782 000 in 2011.

115% of value in 2010 $= \text{PKR } 782\ 000$

1% of value in 2010 $= \frac{\text{PKR } 782\ 000}{115}$

100% of value in 2010 $= \frac{\text{PKR } 782\ 000}{115} \times 100$

$= \text{PKR } 680\ 000$

The value of the condominium was PKR 680 000 in 2010.

14. 75% of value in 2011 = PKR 11 250

$1\% \text{ of value in 2011} = \frac{\text{PKR } 11\ 250}{75}$

$100\% \text{ of value in 2011} = \frac{\text{PKR } 11\ 250}{75} \times 100$

$= \text{PKR } 15\ 000$

The value of the surveying machine was PKR 15 000 in 2011.

75% of value in 2010 = PKR 15 000

$1\% \text{ of value in 2010} = \frac{\text{PKR } 15\ 000}{75}$

$100\% \text{ of value in 2010} = \frac{\text{PKR } 15\ 000}{75} \times 100$

$= \text{PKR } 20\ 000$

The value of the surveying machine was PKR 20 000 in 2010.

15. 105% of value at the end of 2010 = PKR 61 824

$1\% \text{ of value at the end of 2010} = \frac{\text{PKR } 61\ 824}{105}$

$100\% \text{ of value at the end of 2010} = \frac{\text{PKR } 61\ 824}{105} \times 100$

$= \text{PKR } 58\ 880$

The value of the investment portfolio was PKR 58 880 at the end of 2010.

92% of original value = PKR 58 880

$1\% \text{ of original value} = \frac{\text{PKR } 58\ 880}{92}$

$100\% \text{ of original value} = \frac{\text{PKR } 58\ 880}{92} \times 100$

$= \text{PKR } 64\ 000$

The original value of the investment portfolio was PKR 64 000.

16. Let Amirah's height be x m.

108% of Anusha's height = x m

$1\% \text{ of Anusha's height} = \frac{x}{108} \text{ m}$

$100\% \text{ of Anusha's height} = \frac{x}{108} \times 100$

$= \frac{25}{27}x \text{ m}$

Anusha's height is $\frac{25}{27}x$ m.

$\text{Seema's height} = 90\% \times \frac{25}{27}x$

$= \frac{90}{100} \times \frac{25}{27}x$

$= \frac{5}{6}x \text{ m}$

$\text{Required percentage} = \frac{x}{\frac{5}{6}x} \times 100\%$

$= \frac{1}{\frac{5}{6}} \times 100\%$

$= 120\%$

Review Exercise 3

1. $\text{Required percentage} = \frac{1 \text{ m}}{56 \text{ mm}} \times 100\%$

$= \frac{1000 \text{ mm}}{56 \text{ mm}} \times 100\%$

$= \frac{125}{7} \times 100\%$

$= 1785\frac{5}{7}\%$

2. **(i)** Pocket money Maaz receives in a year = 52 × PKR 280

= PKR 14 560

$\text{Savings in a year} = \frac{20}{100} \times \text{PKR } 14\ 560$

= PKR 2912

(ii) Spending in a year = PKR 14 560 – PKR 2912

= PKR 11 648

3. $\frac{a}{4b} = \frac{\frac{30}{100} \times b}{4b}$

$= \frac{\frac{30}{100}}{4}$

$= \frac{3}{40}$

4. $\text{Anusha's percentage score} = \frac{68}{80} \times 100\%$

$= 85\%$

$\text{Seema's percentage score} = \frac{86}{120} \times 100\%$

$= 71\frac{2}{3}\%$

$\text{Rizwan's percentage score} = \frac{120}{150} \times 100\%$

$= 80\%$

∴ Anusha performs the best in her Science test.

5. $\text{Number of apples the vendor has} = \frac{120}{100} \times 120$

$= 144$

60% of number of pears = 120

$1\% \text{ of number of pears} = \frac{120}{60}$

$100\% \text{ of number of pears} = \frac{120}{60} \times 100$

$= 200$

Number of pears the vendor has = 200

Total number of fruits the vendor has = 120 + 144 + 200

= 464

6. 120% of number of pages Kiran reads on the second day = 60

1% of number of pages Kiran reads on the second day = $\frac{60}{120}$

100% of number of pages Kiran reads on the second day

$= \frac{60}{120} \times 100$

$= 50$

Number of pages Kiran reads on the second day = 50

Number of pages in the book = 6 × 50

= 300

7. Percentage of goats left = $\frac{94}{100} \times 86\%$

= 80.84%

80.84% of original number of goats = 8084

1% of original number of goats = $\frac{8084}{80.84}$

100% of original number of goats = $\frac{8084}{80.84} \times 100$

= 10 000

The original number of goats in the village is 10 000.

8. Let Mr Naeem's original salary be PKR x.

Mr Naeem's reduced salary = $\frac{85}{100} \times$ PKR x

= PKR $0.85x$

Required percentage = $\frac{\$x - \$0.85x}{\$0.85x} \times 100\%$

$= \frac{\$0.15x}{\$0.85x} \times 100\%$

$= \frac{0.15}{0.85} \times 100\%$

$= 17\frac{11}{17}\%$

Challenge Yourself

1. Let the number of red jellybeans Amirah moves from Bottle A to Bottle B be x, the number of yellow jellybeans Amirah moves from Bottle A to Bottle B be y.

		Bottle *A*	**Bottle *B***
Before	**Red**	300	150
	Yellow	100	150
After	**Red**	$300 - x$	$150 + x$
	Yellow	$100 - y$	$150 + y$

$\therefore \frac{300 - x}{100 - y} = \frac{80}{20}$

$\frac{300 - x}{100 - y} = \frac{4}{1}$

$300 - x = 4(100 - y)$

$300 - x = 400 - 4y$

$4y - x = 100 \quad — (1)$

$\therefore \frac{150 + x}{150 + y} = \frac{60}{40}$

$\frac{150 + x}{150 + y} = \frac{3}{2}$

$2(150 + x) = 3(150 + y)$

$300 + 2x = 450 + 3y$

$2x - 3y = 150 \quad — (2)$

2 × (1): $8y - 2x = 200 \quad — (3)$

(2) + (3): $5y = 350$

$y = 70$

Substitute $y = 70$ into (1): $4(70) - x = 100$

$280 - x = 100$

$x = 180$

Number of jellybeans Amirah moves from Bottle A to Bottle B

$= x + y$

$= 180 + 70$

$= 250$

2. Percentage of water which is poured from Cup B into Cup A

$= \frac{60}{100} \times 70\%$

$= 42\%$

Percentage of water in Cup A before 60% of solution in Cup A is poured into Cup B

$= 40\% + 42\%$

$= 82\%$

Percentage of water in Cup A after 60% of solution in Cup A is poured into Cup B

$= \frac{40}{100} \times 82\%$

$= 32.8\%$

Chapter 4 Introduction to Sets

TEACHING NOTES

Suggested Approach:

Teachers should not take an abstract approach when introducing the basic set notation, the complement of a set, and the union and intersection of sets. Teachers should try to apply the set language to describe things in daily life to arouse students' interest to learn this topic.

Section 4.1: Introduction to Set Notations

It will be a good idea to introduce this chapter by asking the students to think of sentences that relate to the collection of objects before introducing the mathematical term 'set' which is used to describe any collection of well-defined and distinct objects. It is important to engage the students to discuss the meanings of 'well-defined' and 'distinct' objects (see Class Discussion: Well-defined and Distinct Objects in a Set on page 67 of the textbook) before moving on to the different ways of representing sets as shown on page 67 of the textbook.

Teachers should advise the students that when listing, it will be good if they are to arrange the elements either in ascending order for numbers or alphabetical order for letters or according to the given order.

Students will gain a better understanding of equal sets if they are able to think of a counter-example to justify that the statement: if $n(A) = n(B)$, then $A = B$ is not valid (see Thinking Time on page 69).

Teachers should always use a simple example to introduce the different set notations as well as the meaning of equal and empty sets so that students are able to understand them easily.

Challenge Yourself

Question 1 involves the understanding of the terms, 'element' and 'proper subset'. Teachers may advise the students to use a Venn diagram to have a better understanding of each statement.

WORKED SOLUTIONS

Class Discussion (Well-defined and Distinct Objects in a Set)

1. No, H is not a set as the objects (handsome boys) in the set are not well-defined.
2. $T = \{P_1, P_2\}$ since the 2 identical pens are distinct.
3. $E = \{C, L, E, V, R\}$ since the letter 'E' is not distinct.

Thinking Time (Page 69)

If A and B are two sets such that $n(A) = n(B)$, it may not always be $A = B$. A counter-example is given as follows: Let $A = \{1, 2\}$ and $B = \{3, 4\}$, $n(A) = n(B)$ but $A \neq B$, since the elements in A are different from the elements in B.

Practise Now 1

1. **(a)** $A = \{2, 4, 6, 8\}$
   (b) **(i)** True
   (ii) True
   (iii) False
   (iv) True
   (c) **(i)** $2 \in A$
   (ii) $5 \notin A$
   (iii) $9 \notin A$
   (iv) $6 \in A$
2. $n(B) = 10$

Practise Now 2

(i) $C = \{11, 12, 13, 14, 15, 16, 17\}$
$D = \{10, 11, 12, 13, 14, 15, 16, 17\}$
(ii) No. $10 \in D$ but $10 \notin C$

Practise Now 3

(a) No, a movie may be well-liked by some, but not others.
(b) Yes, it is clear which pupils are fourteen years old.
(c) Yes, it is clear whether someone is an English teacher in the school.

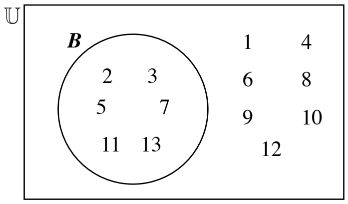
Practise Now 4

(i) $P = \{ \}$
(ii) P and Q are not equal sets, as P is an empty set while Q consists of an element, 0.

Practise Now 5

(i) $\mathbb{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ and
$B = \{2, 3, 5, 7, 11, 13\}$

(ii)



(iii) $B' = \{1, 4, 6, 8, 9, 10, 12\}$
(iv) B' is the set of all integers between 1 and 13 inclusive which are not prime numbers.

Exercise 4A

1. **(a)** $B = \{1, 3, 5, 7, 9\}$
   (b) **(i)** True
   (ii) True
   (iii) False
   (iv) True
2. **(a)** 12
   (b) 8
   (c) –
   (d) –
   (e) 7
   (f) 12
3. **(a)** $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$
   (b) $B = \{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1\}$
   (c) $C = \{2, 4, 6, 8, 10, 12\}$
   (d) $D = \{A\}$
4. **(a)** {A, E, I}
   (b) {red, orange, yellow, green, blue, indigo, violet}
   (c) {9, 18, 27, 36, 45}
   (d) –
   (e) {12, 14, 16, 18, 20, 22}
5. Open ended question; does not require answer.
6. **(a)** Yes, it is clear whether someone has two brothers.
   (b) No, someone may be considered shy to some, but not to others.
   (c) No, an actor may be well-liked by some, but not others.
   (d) No, a dish may be well-liked by some, but not others.
   (e) Yes, it is clear whether a textbook is used in the school.
   (f) No, an actor may be considered most attractive to some, but not others.
7. **(a)** Yes
   (b) Yes
   (c) No
   (d) Yes
   (e) Yes
   (f) Yes
   (g) No
   (h) No
   (i) Yes
   (j) Yes
8. **(a)** $E = \{ \}$. It is an empty set.
   (b) $F = \{ \}$. It is an empty set.

(c) $G = \{\ \}$. It is an empty set.

(d) $H = \{2\}$. It is not an empty set as in contains one element, 2.

9. (a) 2 is the only whole number that is both prime and even.

{2}

The set is a singleton set.

(b) There is no negative integer that is greater than zero.

It is not a singleton set.

(c) The set is not a singleton set.

(d) There are seven days in a week.

The set is not a singleton set.

(e) There are five vowels in English alphabet.

The set is not singleton.

(f) 4 is the only composite number that is less than 5.

The set is a singleton set.

10. (a) D = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

(b) (i) Tuesday $\in D$

(ii) Sunday $\in D$

(iii) March $\notin D$

(iv) Holiday $\notin D$

11. (i) No, 10 is not a perfect square.

(ii) $P = \{4, 9, 16, 25, 36, 49\}$

12. (a) {red, orange, yellow, green, blue, indigo, violet}

(b) {S, Y, M, T, R}

(c) –

(d) {January, June, July}

(e) {11, 13, 15, 17}

(f) {b, c, d, f, g}

(g) {Tuesday, Thursday}

(h) {2, 4, 6, 8, 10, 12}

(i) {February}

13. (a) $M = \{x$: x is an even integer$\}$

(b) $N = \{x$: x is an even integer less than or equal to 8$\}$ or

$N = \{x$: x is an even integer less than 9$\}$

$N = \{x$: x is an even integer less than 10$\}$

(c) $O = \{x$: x is a perfect cube$\}$

(d) $P = \{x$: x is an integer that is a multiple of 5$\}$

(e) $Q = \{x$: x is a digit from the first 5 letters of the alphabet$\}$

14. (a) China; the remaining elements are ASEAN countries

(b) Rubber; the remaining elements are edible fruits

(c) 20; the remaining elements are perfect squares

(d) 75; the remaining elements are perfect cubes

(e) Pie chart; the remaining elements are statistical averages

15. (i) $Q = \{\ \}$, $R = \{1\}$

(ii) $Q = \varnothing$ but $R \neq \varnothing$ as it contains an element, 1.

16. (a) $P = \{\ \}$ or

$Q = \{-2, -1, 0, 1, 2, 3, 4, 5, 6,\}$

(b) Yes, Set Q has only 9 elements.

17. (i) False, as 'c' is an element of the set.

(ii) False, as the word 'car' is not an element of the set.

(iii) False, as {c} is a set, not an element.

(iv) False, as {c, a, r} is a set, not the number of elements in the set.

(v) True, as '5' is an element of the set.

(vi) False, as '4' is not an element of the set.

(vii) False, as the word 'bus' is not an element of the set, only the individual letters are.

(viii) True, as 'b' is an element of the set.

18. (a) True

(b) True

(c) False, as 4 is an even number.

(d) False, as {S, C, O, H, L} is a set, not an element.

(e) False, as 5 is not an even number.

(f) False, as {3} is a set, not an element.

19. (a) $S = \{x$: x is a girl in my current class wearing spectacles$\}$

(b) $T = \{x$: x is a prime number$\}$

(c) $U = \{x$: x is a multiple of 4$\}$

(d) $V = \{x$: x is a multiple of 4 between -8 and 12 inclusive$\}$

20. (i) False, as 0 is an element.

(ii) True

(iii) False, as $\varnothing$ is an element.

(iv) True

21. (a) $R = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

(b) Set U and S are infinite sets.

Review Exercise 4

1. (a) $A = \{1, 3, 5, 7, 9\}$

(b) (i) True

(ii) True

(iii) False

(iv) False

(c) (i) $-3 \notin A$

(ii) $3 \in A$

(iii) $0 \notin A$

(iv) $9 \in A$

2. (a) $B = \{2\}$. It is not an empty set.

(b) C = {Saturday, Sunday}. It is not an empty set.

(c) $D = \varnothing$. It is an empty set.

(d) $E = \varnothing$. It is an empty set.

3. (i) $A = \{1,3\}$

$B = \{4\}$

(ii) Yes, B is a singleton set because it has only one element.

Challenge Yourself

(i) True, since a is an element of the set, S.

(ii) True, since {a} is an element of the set, S.

Chapter 5 Number Patterns and Algebraic Manipulation

TEACHING NOTES

Suggested Approach

Students have done word problems involving number sequences and patterns in primary school. These word problems required the students to recognise simple patterns from various number sequences and determine either the next few terms or a specific term. However, they were not taught to use algebra to solve problems involving number patterns. Teachers can arouse students' interest in this topic by bringing in real-life applications.

Students are still unfamiliar with algebra. Therefore, the teacher should explain algebra from scratch. The learning experiences in the new syllabus specify the use of concrete objects, such as of algebra discs. In addition to the algebra discs showing the numbers 1 and –1 which students have encountered in Chapter 2, algebra discs showing x, $-x$, y and $-y$ are needed. Since many Grade 6 students are still in the concrete operational stage (according to Piaget), the use of algebra discs can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use algebra discs in examinations, and partly because they cannot use algebra discs to manipulate algebraic expressions which consist of algebraic terms that have large or fractional coefficients (see Section 5.2 and 5.3).

Section 5.1: Number Sequences

In primary school, students were only asked how to find the next few terms and a specific term of number sequences but they have not been taught how to state the rule. Teachers can build upon this by getting students to work in pairs to state the rules of number sequences and then write down the next few terms (see Class Discussion: Number Sequences). Students should learn that they can add, subtract, multiply or divide or use a combination of arithmetic operations to get the next term of a number sequence.

Section 5.2: Fundamental Algebra

Teachers should teach students how to use letters to represent numbers and interpret basic algebraic notations such as $ab = a \times b$. Teachers should illustrate the definitions of mathematical terms such as 'algebraic term', 'coefficient', 'algebraic expression' and 'linear expression' using appropriate examples.

In the class discussion on page 82 of the textbook, students are required to use algebraic expressions to express mathematical relationships.

To make learning more interactive, students are given the opportunity to use a spreadsheet to explore the concept of variables (see Investigation: Comparison between Pairs of Expressions). Through this investigation, students should be able to observe that evaluating an algebraic expression means finding the value of the expression when the variables take on certain values. This investigation also provides students with an intuitive sense of the difference between pairs of expressions such as $2n$ and $2 + n$, n^2 and $2n$, and $2n^2$ and $(2n)^2$. Students are expected to give a more rigorous mathematical explanation for the difference between such a pair of expressions in the journal writing on page 85 of the textbook.

Algebra discs cannot be used to add or subtract algebraic terms with large coefficients, so there is a need to help students consolidate what they have learnt in Worked Example 4. For the lower ability students, before going through Worked Example 4(d) and (e), teachers should revisit the procedure for simplifying ordinary numerical fractions, e.g. $\frac{1}{2} + \frac{1}{3}$.

Section 5.3: Expansion and Simplification of Linear Expressions

The idea of flipping over a disc to obtain the negative of a number or variable, e.g. $-(-x) = x$, is needed to teach students how to obtain the negative of a linear expression. Algebra discs cannot be used to manipulate algebraic expressions which consist of algebraic terms that have large coefficients, so there is a need to help students consolidate what they have learnt in the class discussion on page 93 of the textbook by moving away from the 'concrete' to the following 'abstract' concept:

Distributive Law: $a(b + c) = ab + ac$

Teachers should emphasise the importance of the rules by which operations are performed when an algebraic expression involves brackets by using the thinking time on page 96 of the textbook.

WORKED SOLUTIONS

Class Discussion (Number Sequences)

1.

	Sequence	Rule
Positive even numbers	2, 4, 6, 8, 10, 12, 14, … (+2 +2 +2 +2 +2 +2)	Start with 2, then add 2 to each term to get the next term.
Positive odd numbers	1, 3, 5, 7, 9, 11, 13, … (+2 +2 +2 +2 +2 +2)	Start with 1, then add 2 to each term to get the next term.
Multiples of 3	3, 6, 9, 12, 15, 18, 21, … (+3 +3 +3 +3 +3 +3)	Start with 3, then add 3 to each term to get the next term.
Powers of 2	1, 2, 4, 8, 16, 32, 64, … (×2 ×2 ×2 ×2 ×2 ×2)	Start with 1, then multiply each term by 2 to get the next term.
Powers of 3	1, 3, 9, 27, 81, 243, 729, … (×3 ×3 ×3 ×3 ×3 ×3)	Start with 1, then multiply each term by 3 to get the next term.

Table 5.1

2. The sequence of positive odd numbers can be obtained by subtracting 1 from each term of the sequence 2, 4, 6, 8, 10, ….

Teachers may wish to note that there are other possible answers to this question.

3. **(a)** Rule: Find the square of the position of each term. The next two terms are 36 and 49.

(b) Rule: Find the cube of the position of each term. The next two terms are 216 and 343.

Class Discussion (Expressing Mathematical Relationships using Algebra)

4.

	In words	Algebraic expression
(a)	Sum of $2x$ and $3z$	$2x + 3z$
(b)	Product of x and $7y$	$7xy$
(c)	Divide $3ab$ by $2c$	$\frac{3ab}{2c}$
(d)	Subtract $6q$ from $10z$	$10z - 6q$
(e)	Subtract the product of x and y from the sum of p and q	$(p + q) - xy$
(f)	Divide the sum of 3 and y by 5	$\frac{3+y}{5}$
(g)	Subtract the product of 2 and c from the positive square root of b	$\sqrt{b} - 2c$
(h)	There are three times as many girls as boys in a school. Find an expression, in terms of x, for the total number of students in the school, where x represents the number of boys in the school.	It is given that x represents the number of boys. $\therefore$ $3x$ represents the number of girls. Total number of students $= x + 3x$ $= 4x$
(i)	The age of Nora's father is thrice hers. The age of Nora's brother is 5 years more than hers. Find an expression, in terms of y, for the sum of their ages, where y represents Nora's age.	It is given that y represents Nora's age. $\therefore$ Nora's father is $3y$ years old. Nora's brother is $(y + 5)$ years old. Sum of their ages $= y + 3y + y + 5$ $= (5y + 5)$ years
(j)	The length is three times as long as the breadth of the rectangle. Find an expression, in terms of b, for the perimeter and the area of the rectangle, where b represents the breadth of the rectangle.	It is given that b represents the breadth of the rectangle in m. $3b$ represents the length of the rectangle in m. Perimeter of rectangle $= 2(3b + b)$ $= 2(4b)$ $= 8b$ m Area of rectangle $= 3b \times b$ $= 3b^2$ m^2

Table 5.4

Investigation (Comparison between Pairs of Expressions)

5.

	A	B	C	D	E	F
1						
2	n	$2n$	$2 + n$	n^2	$2n^2$	$(2n)^2$
3	1	2				
4	2	4				
5	3	6				
6	4	8				
7	5	10				

(i) The value of $2n$ changes as n changes.

(ii) We multiply the given value of n by 2 to obtain the corresponding value of $2n$.

(iii) When $n = 8$,
$2n = 2 \times 8$
$= 16$
When $n = 9$,
$2n = 2 \times 9$
$= 18$
When $n = 10$,
$2n = 2 \times 10$
$= 20$

6.

	A	B	C	D	E	F
1						
2	n	$2n$	$2 + n$	n^2	$2n^2$	$(2n)^2$
3	1	2	3	1	2	4
4	2	4	4	4	8	16
5	3	6	5	9	18	36
6	4	8	6	16	32	64
7	5	10	7	25	50	100

7. • $2n$ and $2 + n$

Referring to columns B and C on the spreadsheet, the expressions $2n$ and $2 + n$ are equal only when $n = 2$. When $n < 2$, $2n < 2 + n$. When $n > 2$, $2n > 2 + n$.

• n^2 and $2n$

Referring to columns B and D on the spreadsheet, the expressions n^2 and $2n$ are equal when $n = 2$. By observation, they are also equal when $n = 0$. When $n < 0$ or $n > 2$, $n^2 > 2n$. When $0 < x < 2$, $n^2 < 2n$.

• $2n^2$ and $(2n)^2$

By observation, the expressions $2n^2$ and $(2n)^2$ are equal when $n = 0$. For any value of $n \neq 0$, $(2n)^2 > 2n^2$.

Journal Writing (Page 85)

By observation, the expressions $5 + n$ and $5n$ are equal only when $n = 1\frac{1}{4}$. When $n < 1\frac{1}{4}$, $5n < 5 + n$. When $n > 1\frac{1}{4}$, $5n > 5 + n$.

Class Discussion (The Distributive Law)

1. (a) $2(-x - 4) = -2x - 8$
   (b) $-2(-x - 4) = 2x + 8$
   (c) $3(y - 2x) = 3y - 6x$
   (d) $-3(y - 2x) = -3y + 6x$

2. $a(b + c) = ab + ac$

Thinking Time (Page 96)

$$
\begin{aligned}
-(x - 5) + 6x - (7x - 2) + 12 &= -x + 5 + 6x - 7x + 2 + 12\\
&= -x + 6x - 7x + 5 + 2 + 12\\
&= -2x + 19
\end{aligned}
$$

Possible ways:

• $$
\begin{aligned}
-(x - 5) + 6x - 7x - (2 + 12) &= -(x - 5) + 6x - 7x - 14\\
&= -x + 5 + 6x - 7x - 14\\
&= -x + 6x - 7x + 5 - 14\\
&= -2x - 9
\end{aligned}
$$

• $$
\begin{aligned}
-x - (5 + 6x) - (7x - 2) + 12 &= -x - 5 - 6x - 7x + 2 + 12\\
&= -x - 6x - 7x - 5 + 2 + 12\\
&= -14x + 9
\end{aligned}
$$

• $$
\begin{aligned}
-x - (5 + 6x) - 7x - (2 + 12) &= -x - (5 + 6x) - 7x - 14\\
&= -x - 5 - 6x - 7x - 14\\
&= -x - 6x - 7x - 5 - 14\\
&= -14x - 19
\end{aligned}
$$

Practise Now 1

1. (a) Rule: Add 5 to each term to get the next term. The next two terms are 28 and 33.
   (b) Rule: Subtract 6 from each term to get the next term. The next two terms are –50 and –56.
   (c) Rule: Multiply each term by 3 to get the next term. The next two terms are 1215 and 3645.
   (d) Rule: Divide each term by –3 to get the next term. The next two terms are –18 and 6.

2. (a) 22, 29
   (b) 15, 11

Practise Now 2

1. (a) $$
\begin{aligned}
5y - 4x &= 5(4) - 4(-2)\\
&= 20 + 8\\
&= 28
\end{aligned}
$$

   (b) $$
\begin{aligned}
\frac{1}{x} - y + 3 &= \frac{1}{-2} - 4 + 3\\
&= -\frac{1}{2} - 4 + 3\\
&= -4\frac{1}{2} + 3\\
&= -1\frac{1}{2}
\end{aligned}
$$

2. $$
\begin{aligned}
p^2 + 3q^2 &= \left(-\frac{1}{2}\right)^2 + 3(-2)^2\\
&= \frac{1}{4} + 3(4)\\
&= \frac{1}{4} + 12\\
&= 12\frac{1}{4}
\end{aligned}
$$

Practise Now 3

(a) $3x + 4x = 7x$
(b) $3x + (-4x) = -x$
(c) $-3x + 4x = x$
(d) $-3x + (-4x) = -7x$

Practise Now 4

(a) $4x - 3x = x$
(b) $4x - (-3x) = 4x + 3x$
$= 7x$
(c) $-4x - 3x = -7x$
(d) $-4x - (-3x) = -4x + 3x$
$= -x$

Practise Now 5

(a) $$
\begin{aligned}
x + 2 + 5x - 4 &= x + 5x + 2 - 4\\
&= 6x - 2
\end{aligned}
$$

(b) $$
\begin{aligned}
2x + (-3) - 3x + 5 &= 2x - 3x + (-3) + 5\\
&= -x + 2
\end{aligned}
$$

(c) $$
\begin{aligned}
-x - y - (-2x) + 4y &= -x - y + 2x + 4y\\
&= -x + 2x - y + 4y\\
&= x + 3y
\end{aligned}
$$

(d) $$
\begin{aligned}
-3x - 7y + (-2y) - (-4x) &= -3x - 7y + (-2y) + 4x\\
&= -3x + 4x - 7y + (-2y)\\
&= x - 9y
\end{aligned}
$$

Practise Now 6

1. **(a)** $$\begin{aligned} 2x - 5y + 4y + 8x &= 2x + 8x - 5y + 4y \\ &= 10x - y \end{aligned}$$

(b) $$\begin{aligned} 11x - (-5y) - 14x - 2y &= 11x + 5y - 14x - 2y \\ &= 11x - 14x + 5y - 2y \\ &= -3x + 3y \end{aligned}$$

(c) $$\begin{aligned} -9x - (-y) + (-3x) - 7y &= -9x + y - 3x - 7y \\ &= -9x - 3x + y - 7y \\ &= -12x - 6y \end{aligned}$$

(d) $$\begin{aligned} \frac{1}{2}x - \frac{1}{3}x &= \frac{3}{6}x - \frac{2}{6}x \\ &= \frac{1}{6}x \end{aligned}$$

(e) $$\begin{aligned} \frac{7}{4}y - \frac{5}{8}y &= \frac{14}{8}y - \frac{5}{8}y \\ &= \frac{9}{8}y \end{aligned}$$

2. **(i)** $$\begin{aligned} 2p - 5q + 7r - 4p + 2q - 3r &= 2p - 4p - 5q + 2q + 7r - 3r \\ &= -2p - 3q + 4r \end{aligned}$$

(ii) When $p = \frac{1}{2}$, $q = -\frac{1}{3}$, $r = 4$,

$$\begin{aligned} -2p - 3q + 4r &= -2\ \ \frac{1}{2}\ \ - 3\ \ -\frac{1}{3}\ \ + 4(4) \\ &= -1 + 1 + 16 \\ &= 0 + 16 \\ &= 16 \end{aligned}$$

Practise Now 7

(a) $-(3x + 2) = -3x - 2$
(b) $-(3x - 2) = -3x + 2$
(c) $-(-3x - 2) = 3x + 2$
(d) $-(2x + y - 4) = -2x - y + 4$

Practise Now 8

(a) $$\begin{aligned} x + 1 + [-(3x - 1)] &= x + 1 - 3x + 1 \\ &= x - 3x + 1 + 1 \\ &= -2x + 2 \end{aligned}$$

(b) $$\begin{aligned} 5x - 3 + [-(4x + 1)] &= 5x - 3 - 4x - 1 \\ &= 5x - 4x - 3 - 1 \\ &= x - 4 \end{aligned}$$

(c) $$\begin{aligned} 3x + 2y + [-(-y + 2x)] &= 3x + 2y + y - 2x \\ &= 3x - 2x + 2y + y \\ &= x + 3y \end{aligned}$$

(d) $$\begin{aligned} -4x + 2y + [-(-x - 5y)] &= -4x + 2y + x + 5y \\ &= -4x + x + 2y + 5y \\ &= -3x + 7y \end{aligned}$$

Practise Now 9

(a) $3(5x) = 15x$
(b) $3(-5x) = -15x$
(c) $-3(5x) = -15x$
(d) $-3(-5x) = 15x$

Practise Now 10

(a) $3(x + 2) = 3x + 6$
(b) $-5(x - 4y) = -5x + 20y$
(c) $$\begin{aligned} -a(x + 2y) &= -ax - a(2y) \\ &= -ax - 2ay \end{aligned}$$

Practise Now 11

(a) $$\begin{aligned} x + 7 + 3(x - 2) &= x + 7 + 3x - 6 \\ &= x + 3x + 7 - 6 \\ &= 4x + 1 \end{aligned}$$

(b) $$\begin{aligned} 3(x + 2) + 2(-2x + 1) &= 3x + 6 - 4x + 2 \\ &= 3x - 4x + 6 + 2 \\ &= -x + 8 \end{aligned}$$

(c) $$\begin{aligned} 2(-x - y) - (2x - y) &= -2x - 2y - 2x + y \\ &= -2x - 2x - 2y + y \\ &= -4x - y \end{aligned}$$

(d) $$\begin{aligned} -(x + 4y) - 2(3x - y) &= -x - 4y - 6x + 2y \\ &= -x - 6x - 4y + 2y \\ &= -7x - 2y \end{aligned}$$

Practise Now 12

1. **(a)** $$\begin{aligned} 6(4x + y) + 2(x - y) &= 24x + 6y + 2x - 2y \\ &= 24x + 2x + 6y - 2y \\ &= 26x + 4y \end{aligned}$$

(b) $$\begin{aligned} x - [y - 3(2x - y)] &= x - (y - 6x + 3y) \\ &= x - (-6x + y + 3y) \\ &= x - (-6x + 4y) \\ &= x + 6x - 4y \\ &= 7x - 4y \end{aligned}$$

(c) $$\begin{aligned} 7x - 2[3(x - 2) - 2(x - 5)] &= 7x - 2(3x - 6 - 2x + 10) \\ &= 7x - 2(3x - 2x - 6 + 10) \\ &= 7x - 2(x + 4) \\ &= 7x - 2x - 8 \\ &= 5x - 8 \end{aligned}$$

2. **(i)** Maaz's present age = $(p + 5)$ years

(ii) Bilal's present age $= 3(p + 5)$
$= (3p + 15)$ years

(iii) Sum of their ages in 6 years' time

$$\begin{aligned} &= p + p + 5 + 3p + 15 + 3 \times 6 \\ &= p + p + 5 + 3p + 15 + 18 \\ &= p + p + 3p + 5 + 15 + 18 \\ &= (5p + 38) \text{ years} \end{aligned}$$

(iv) Sum of their ages 3 years ago $$\begin{aligned} &= p + p + 5 + 3p + 15 - 3 \times 3 \\ &= p + p + 5 + 3p + 15 - 9 \\ &= p + p + 3p + 5 + 15 - 9 \\ &= (5p + 11) \text{ years} \end{aligned}$$

Alternatively,

Sum of their ages 3 years ago $$\begin{aligned} &= 5p + 38 - 3 \times 9 \\ &= 5p + 38 - 27 \\ &= (5p + 11) \text{ years} \end{aligned}$$

Exercise 5A

1. (a) Rule: Add 5 to each term to get the next term. The next two terms are 39 and 44.
   (b) Rule: Subtract 8 from each term to get the next term. The next two terms are 40 and 32.
   (c) Rule: Multiply each term by 2 to get the next term. The next two terms are 384 and 768.
   (d) Rule: Divide each term by 2 to get the next term. The next two terms are 50 and 25.
   (e) Rule: Divide each term by –4 to get the next term. The next two terms are 16 and –4.
   (f) Rule: Multiply each term by –2 to get the next term. The next two terms are –288 and 576.
   (g) Rule: Subtract 7 from each term to get the next term. The next two terms are –87 and –94.
   (h) Rule: Add 10 to each term to get the next term. The next two terms are –50 and –40.
   (i) Rule: Add 10 to each term to get the next term. The next two terms are 50 and 60.
   (j) Rule: Add 7 to each term to get the next term. The next two terms are 80 and 87.
   (k) Rule: Multiply each term by 3 to get the next term. The next two terms are 324 and 972.

2.

Minutes	1	2	3	4	5	6	7
Distance covered (metres)	250	270	290	310	330	350	370

The car will cover 370 m in 7th minute

3.

Rows	1	2	3	4	5	6	7	8	9
Seats	32	40	48	56	64	72	80	88	96

There will be 96 seats in 9th row.

4.

Story	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of people	53	60	67	74	81	88	95	102	109	116	123	130	137	145	151

151 people will be living on top most floor.

5.

Month	1	2	3	4	5
Amount (PKR)	5000	6500	8000	9500	11000

She will have Rs 11,000 after 5 months.

6. (a) $6n$
   6(1), 6(2), 6(3), 6(4), 6(5)
   5, 12, 18, 24, 30
   (b) $n + 3$
   1+3, 2+3, 3+3, 4+3, 5+3
   4, 5, 6, 7, 8
   (c) $2n + 1$
   2(1)+1, 2(2)+1, 2(3)+1, 2(4)+1, 2(5)+1
   3, 5, 7, 9, 11
   (d) $4n – 1$
   4(1)-1, 4(2)-1, 4(3)-1, 4(4)-1, 4(5)-1
   3, 9, 11, 13, 15, 19
   (e) $2n + 5$
   2(1)+5, 2(2)+5, 2(3)+5, 2(4)+5, 2(5)+5
   7, 9, 11, 13, 15
   (f) $6n – 3$
   6(1)-3, 6(2)-3, 6(3)-3, 6(4)-3, 6(5)-3
   3, 9, 15, 21, 27

7. (a) 9, 15
   (b) 12, 8
   (c) –33, –32
   (d) 88, 85
   (e) 21, 28

8.

Days	1	2	3	4	5	6
Number of boys	8	13	18	23	28	33

23, 28, and 33 students registered in next three days, respectively.

9.

Rows	1	2	3	4	5	6	7
Number of people	75	87	99	111	123	135	147

There are 147 people in 7th row.

10. (a) –67, –131
   (b) 8, 13
   (c) 144, 196
   (d) –216, 343
   (e) 81, 243

11.

Days	1	2	3	4
Duration of swimming (minutes)	35	35 + 10 = 45	45 + 10 = 55	55 + 10 = 65

65 minutes = 1 hour 5 minutes
He require 4 days to practice 1 hour 5 minutes

Exercise 5B

1. (a) $ab + 5y$ (b) $f^3 – 3$
   (c) $6kq$ (d) $\frac{2w}{3xy}$
   (e) $3x – 4\sqrt{z}$ (f) $\frac{2p}{5q}$

2. (a) $4x – 7y = 4(6) – 7(–4)$
   $= 24 + 28$
   $= 52$

(b) $\frac{5x}{3y} + x = \frac{5(6)}{3(-4)} + 6$

$= \frac{30}{-12} + 6$

$= -2\frac{1}{2} + 6$

$= 3\frac{1}{2}$

(c) $2x^2 - y^3 = 2(6)^2 - (-4)^3$

$= 72 - (-64)$

$= 72 + 64$

$= 136$

(d) $3x + \frac{x}{y} - y^2 = 3(6) + \frac{6}{-4} - (-4)^2$

$= 18 - 1\frac{1}{2} - 16$

$= 16\frac{1}{2} - 16$

$= \frac{1}{2}$

3. **(a)** $a(3c - b) = 3[3(6) - (-5)]$

$= 3(18 + 5)$

$= 3(23)$

$= 69$

(b) $ab^2 - ac = 3(-5)^2 - 3(6)$

$= 3(25) - 18$

$= 75 - 18$

$= 57$

(c) $\frac{b}{a} - \frac{c}{b} = \frac{-5}{3} - \frac{6}{-5}$

$= -1\frac{2}{3} + 1\frac{1}{5}$

$= -\frac{7}{15}$

(d) $\frac{b+c}{a} + \frac{a+c}{b} = \frac{-5+6}{3} + \frac{3+6}{-5}$

$= \frac{1}{3} + \frac{9}{-5}$

$= \frac{1}{3} - 1\frac{4}{5}$

$= -1\frac{7}{15}$

4. **(a)** $5x + 22 - 6x - 23 = 5x - 6x + 22 - 23$

$= -x - 1$

(b) $x + 3y + 6x + 4y = x + 6x + 3y + 4y$

$= 7x + 7y$

(c) $6xy + 13x - 2yx - 5x = 6xy - 2yx + 13x - 5x$

$= 4xy + 8x$

(d) $6x - 20y + 7z - 8x + 25y - 11z$

$= 6x - 8x - 20y + 25y + 7z - 11z$

$= -2x + 5y - 4z$

5. **(a)** Required answer $= 2x + 4y + (-5y)$

$= 2x + 4y - 5y$

$= 2x - y$

(b) Required answer $= -b - 4a + 7b - 6a$

$= -4a - 6a - b + 7b$

$= -10a + 6b$

(c) Required answer $= 6d - 4c + (-7c + 6d)$

$= 6d - 4c - 7c + 6d$

$= -4c - 7c + 6d + 6d$

$= -11c + 12d$

(d) Required answer $= 3pq - 6hk + (-3qp + 14kh)$

$= 3pq - 6hk - 3qp + 14kh$

$= 3pq - 3qp - 6hk + 14kh$

$= 8hk$

6. **(a)** $(a + b)^2 - \sqrt[3]{3xy}$

(b) Total value $= (20x + 500y)$ paisas

7. **(a)** $\frac{3a-b}{2c} + \frac{3a-c}{c-b} = \frac{3(3)-(-4)}{2(-2)} + \frac{3(3)-(-2)}{-2-(-4)}$

$= \frac{9+4}{-4} + \frac{9+2}{-2+4}$

$= \frac{13}{-4} + \frac{11}{2}$

$= -3\frac{1}{4} + 5\frac{1}{2}$

$= 2\frac{1}{4}$

(b) $\frac{2c-a}{3c+b} - \frac{5a+4c}{c-a} = \frac{2(-2)-3}{3(-2)+(-4)} - \frac{5(3)+4(-2)}{-2-3}$

$= \frac{-4-3}{-6-4} - \frac{15-8}{-5}$

$= \frac{-7}{-10} - \frac{7}{-5}$

$= \frac{7}{10} + 1\frac{2}{5}$

$= 2\frac{1}{10}$

(c) $\frac{a+b+2c}{3c-a-b} - \frac{5c}{4b} = \frac{3+(-4)+2(-2)}{3(-2)-3-(-4)} - \frac{5(-2)}{4(-4)}$

$= \frac{3-4-4}{-6-3+4} - \frac{-10}{-16}$

$= \frac{-5}{-5} - \frac{5}{8}$

$= 1 - \frac{5}{8}$

$= \frac{3}{8}$

(d) $\frac{b-c}{3c+4b} \div \frac{bc}{a} + \frac{ac}{b}$

$= \frac{-4-(-2)}{3(-2)+4(-4)} \div \frac{(-4)(-2)}{3} + \frac{3(-2)}{-4}$

$= \frac{-4+2}{-6-16} \div \frac{8}{3} + \frac{-6}{-4}$

$= \frac{-2}{-22} \div 2\frac{2}{3} + 1\frac{1}{2}$

$= \frac{1}{11} \div 4\frac{1}{6}$

$= \frac{6}{275}$

8. **(a)** $15x + (-7y) + (-18x) + 4y = 15x - 7y - 18x + 4y$
$= 15x - 18x - 7y + 4y$
$= -3x - 3y$

(b) $-3x + (-5y) - (-10y) - 7x = -3x - 5y + 10y - 7x$
$= -3x - 7x - 5y + 10y$
$= -10x + 5y$

(c) $9x - (-2y) - 8x - (-12y) = 9x + 2y - 8x + 12y$
$= 9x - 8x + 2y + 12y$
$= x + 14y$

(d) $-7x - (-15y) - (-2x) + (-6y) = -7x + 15y + 2x - 6y$
$= -7x + 2x + 15y - 6y$
$= -5x + 9y$

9. **(a)** $\frac{1}{4}x + \frac{1}{3}x = \frac{3}{12}x + \frac{4}{12}x$
$= \frac{7}{12}x$

(b) $\frac{2}{5}y - \frac{1}{3}y = \frac{6}{15}y - \frac{5}{15}y$
$= \frac{1}{15}y$

(c) $-\frac{3}{7}a + \frac{3}{5}a = -\frac{15}{35}a + \frac{21}{35}a$
$= \frac{6}{35}a$

(d) $\frac{9}{4}b - \frac{4}{3}b = \frac{27}{12}b - \frac{16}{12}b$
$= \frac{11}{12}b$

10. **(i)** $3p + (-q) - 7r - (-8p) - q + 2r = 3p - q - 7r + 8p - q + 2r$
$= 3p + 8p - q - q - 7r + 2r$
$= 11p - 2q - 5r$

(ii) When $p = 2$, $q = -1\frac{1}{2}$, $r = -5$,

$11p - 2q - 5r = 11(2) - 2 \quad -1\frac{1}{2} \quad - 5(-5)$
$= 22 + 3 + 25$
$= 25 + 25$
$= 50$

11. **(i)** Saad's age 5 years later = $(12m + 5)$ years

(ii) Present age of Saad's son $= 12m - 9m$
$= 3m$ years

Age of Saad's son 5 years later = $(3m + 5)$ years

Sum of their ages in 5 years' time $= 12m + 5 + 3m + 5$
$= 12m + 3m + 5 + 5$
$= (15m + 10)$ years

12. Amount of money Anusha had at first
$= 8 \times$ PKR $w + 7 \times$ PKR $m +$ PKR $(3w + 5m)$
$=$ PKR $8w +$ PKR $7m +$ PKR $(3w + 5m)$
$=$ PKR $(8w + 3w + 7m + 5m)$
$=$ PKR $(11w + 12m)$

13. **(a)** Number of people who order plain prata $= \frac{5}{2}a$

(b) Number of people who order egg prata $= \frac{2}{5}b$

(c) Number of people who order egg prata $= \frac{2}{7}c$

Exercise 5C

1. **(a)** $-(x + 5) = -x - 5$

(b) $-(4 - x) = -4 + x$

(c) $2(3y + 7) = 6y + 14$

(d) $8(2y - 5) = 16y - 40$

(e) $8(3a - 4b) = 24a - 32b$

(f) $-3(c + 6) = -3c - 18$

(g) $-4(d - 6) = -4d + 24$

(h) $2a(x - y) = 2ax - 2ay$

2. **(a)** $5(a + 2b) - 3b = 5a + 10b - 3b$
$= 5a + 7b$

(b) $7(p + 10q) + 2(6p + 7q) = 7p + 70q + 12p + 14q$
$= 7p + 12p + 70q + 14q$
$= 19p + 84q$

(c) $a + 3b - (5a - 4b) = a + 3b - 5a + 4b$
$= a - 5a + 3b + 4b$
$= -4a + 7b$

(d) $x + 3(2x - 3y + z) + 7z = x + 6x - 9y + 3z + 7z$
$= 7x - 9y + 10z$

3. Present age of Hussain's uncle $= 4(x + 5)$
$= (4x + 20)$ years

4. Total cost $= 4x + 6(x - y)$
$= 4x + 6x - 6y$
$= (10x - 6y)$ paisas

5. Total cost of candies Bina bought
$= 7 \times$ PKR $x + n \times$ PKR $12 + (2n + 1) \times$ PKR $15 + 4 \times$ PKR $3x$
$=$ PKR $7x +$ PKR $12n +$ PKR $15(2n + 1) +$ PKR $12x$
$=$ PKR $7x +$ PKR $12n +$ PKR $(30n + 15) +$ PKR $12x$
$=$ PKR $(7x + 12n + 30n + 15 + 12x)$
$=$ PKR $(7x + 12x + 12n + 30n + 15)$
$=$ PKR $(19x + 42n + 15)$

6. **(a)** $4u - 3(2u - 5v) = 4u - 6u + 15v$
$= -2u + 15v$

(b) $-2a - 3(a - b) = -2a - 3a + 3b$
$= -5a + 3b$

(c) $7m - 2n - 2(3n - 2m) = 7m - 2n - 6n + 4m$
$= 7m + 4m - 2n - 6n$
$= 11m - 8n$

(d) $5(2x + 4) - 3(-6 - x) = 10x + 20 + 18 + 3x$
$= 10x + 3x + 20 + 18$
$= 13x + 38$

(e) $-4(a - 3b) - 5(a - 3b) = -4a + 12b - 5a + 15b$
$= -4a - 5a + 12b + 15b$
$= -9a + 27b$

(f) $5(3p - 2q) - 2(3p + 2q) = 15p - 10q - 6p - 4q$
$= 15p - 6p - 10q - 4q$
$= 9p - 14q$

(g) $x + y - 2(3x - 4y + 3) = x + y - 6x + 8y - 6$
$= x - 6x + y + 8y - 6$
$= -5x + 9y - 6$

(h) $3(p - 2q) - 4(2p - 3q - 5) = 3p - 6q - 8p + 12q + 20$
$= 3p - 8p - 6q + 12q + 20$
$= -5p + 6q + 20$

(i) $9(2a + 4b - 7c) - 4(b - c) - 7(-c - 4b)$
$= 18a + 36b - 63c - 4b + 4c + 7c + 28b$
$= 18a + 36b - 4b + 28b - 63c + 4c + 7c$
$= 18a + 60b - 52c$

(j) $-4[5(2x + 3y) - 4(x + 2y)] = -4(10x + 15y - 4x - 8y)$
$= -4(10x - 4x + 15y - 8y)$
$= -4(6x + 7y)$
$= -24x - 28y$

7. **(a)** Required answer $= 2x - 5 - (-6x - 3)$
$= 2x - 5 + 6x + 3$
$= 2x + 6x - 5 + 3$
$= 8x - 2$

(b) Required answer $= 10x - 2y + z - (6x - y + 5z)$
$= 10x - 2y + z - 6x + y - 5z$
$= 10x - 6x - 2y + y + z - 5z$
$= 4x - y - 4z$

(c) Required answer $= -4p - 4q + 15sr - (8p + 9q - 5rs)$
$= -4p - 4q + 15sr - 8p - 9q + 5rs$
$= -4p - 8p - 4q - 9q + 15sr + 5rs$
$= -12p - 13q + 20rs$

(d) Required answer $= 10a - b - 4c - 8d - (8a - 3b + 5c - 4d)$
$= 10a - b - 4c - 8d - 8a + 3b - 5c + 4d$
$= 10a - 8a - b + 3b - 4c - 5c - 8d + 4d$
$= 2a + 2b - 9c - 4d$

8. **(a)** $-2\{3a - 4[a - (2 + a)]\} = -2[3a - 4(a - 2 - a)]$
$= -2[3a - 4(-2)]$
$= -2(3a + 8)$
$= -6a - 16$

(b) $5\{3c - [d - 2(c + d)]\} = 5[3c - (d - 2c - 2d)]$
$= 5[3c - (-2c - d)]$
$= 5(3c + 2c + d)$
$= 5(5c + d)$
$= 25c + 5d$

9. Average monthly salary of the female employees

$= \text{PKR } \dfrac{20000(m + f) - m(b + 200)}{f}$

$= \text{PKR } \dfrac{20000m + 2000f - mb - 200m}{f}$

$= \text{PKR } \dfrac{20000m - 200m - mb + 2000f}{f}$

$= \text{PKR } \dfrac{18000m - mb + 2000f}{f}$

Review Exercise 5

1. **(a)** $-9 \quad -9 \quad -9 \quad -9 \quad -9 \quad -9$
$98,\ 89,\ 80,\ 71,\ 62,\ \underline{53},\ \underline{44}$

(b) $+2 \quad +4 \quad +6 \quad +8 \quad +10 \quad +12$
$-2,\ 0,\ 4,\ 10,\ 18,\ \underline{28},\ \underline{30}$

(c) $\div 3 \quad \div 3 \quad \div 3 \quad \div 3 \quad \div 3 \quad \div 3$
$9,\ 3,\ 1,\ \frac{1}{3},\ \frac{1}{9},\ \frac{1}{27},\ \frac{1}{81}$

(d) $+8 \quad +16 \quad +24 \quad +32 \quad +40 \quad +48$
$1,\ 9,\ 25,\ 49,\ 81,\ \underline{121},\ \underline{169}$

2. **(a)** $4a + 5b = 4(-2) + 5(7)$
$= -8 + 35$
$= 27$

(b) $2a^2 = 2(-2)^2$
$= 8$

(c) $(2a)^2 = [2(-2)]^2$
$= (-4)^2$
$= 16$

(d) $a(b - a) = (-2)[7 - (-2)]$
$= (-2)(7 + 2)$
$= (-2)(9)$
$= -18$

(e) $b - a^2 = 7 - (-2)^2$
$= 7 - 4$
$= 3$

(f) $(b - a)^2 = [7 - (-2)]^2$
$= (7 + 2)^2$
$= 9^2$
$= 81$

3. $$\frac{3x - 5y^2 - 2xyz}{\frac{x}{y} - \frac{y^2}{z}} = \frac{3(3) - 5(-4)^2 - 2(3)(-4)(2)}{\frac{3}{-4} - \frac{(-4)^2}{2}}$$

$$= \frac{9 - 80 + 48}{-\frac{3}{4} - \frac{16}{2}}$$

$$= \frac{-23}{-\frac{3}{4} - \frac{32}{4}}$$

$$= \frac{-23}{-\frac{35}{4}}$$

$$= 2\frac{22}{35}$$

4. **(a)** $3ab - 5xy + 4ab + 2yx = 3ab + 4ab - 5xy + 2yx$
$= 7ab - 3xy$

(b) $4(3p - 5q) + 6(2q - 5p) = 12p - 20q + 12q - 30p$
$= 12p - 30p - 20q + 12q$
$= -18p - 8q$

(c) $2a + 3[a - (b - a)] + 7(2b - a) = 2a + 3(a - b + a) + 7(2b - a)$
$= 2a + 3(a + a - b) + 7(2b - a)$
$= 2a + 3(2a - b) + 7(2b - a)$
$= 2a + 6a - 3b + 14b - 7a$
$= 2a + 6a - 7a - 3b + 14b$
$= a + 11b$

(d) $-2[3x - (4x - 5y) - 2(3x - 4y)] = -2(3x - 4x + 5y - 6x + 8y)$
$= -2(3x - 4x - 6x + 5y + 8y)$
$= -2(-7x + 13y)$
$= 14x - 26y$

(e) $4\{h - 3[f - 6(f - h)]\} = 4[h - 3(f - 6f + 6h)]$
$= 4[h - 3(-5f + 6h)]$
$= 4(h + 15f - 18h)$
$= 4(15f + h - 18h)$
$= 4(15f - 17h)$
$= 60f - 68h$

(f) $5(x + 5y) - \{2x - [3x - 3(x - 2y) + y]\}$
$= 5(x + 5y) - [2x - (3x - 3x + 6y + y)]$
$= 5(x + 5y) - (2x - 7y)$
$= 5x + 25y - 2x + 7y$
$= 5x - 2x + 25y + 7y$
$= 3x + 32y$

5. (a) Total value of 5 rupee-coins $=$ PKR $5x$
(b) Total value of 10 rupee-coins $=$ PKR $(3x \times 10)$
$=$ PKR $30x$
(c) Number of 10 rupee-coins $= \frac{3}{7}x$
Total value of coins $=$ PKR $5x + \frac{3}{7}x \times 10$
$=$ PKR $5x + \frac{30}{7}x$
$=$ PKR $\frac{35}{7}x + \frac{30}{7}x$
$=$ PKR $\frac{65}{7}x$

3. Let the two numbers be $\overline{xy}$ and $\overline{xz}$, where $y + z = 10$.
$\overline{xy} = 10x + y$
$\overline{xz} = 10x + z$
$\therefore\ \overline{xy} \times \overline{xz} = (10x + y)(10x + z)$
$= 10x(10x + z) + y(10x + z)$
$= 100x^2 + 10xz + 10xy + yz$
$= 100x^2 + 10x(y + z) + yz$
$= 100x^2 + 10x(10) + yz$
$= 100x^2 + 100x + yz$
$= 100x(x + 1) + yz$

Challenge Yourself

1. Let the number of heads up in the pile of 5 be x.
Then the number of tails up in the pile of 5 is $5 - x$,
the number of heads up in the pile of 7 is $5 - x$.
After the teacher flips over all the coins in the pile of 5,
the number of heads up in that pile is $5 - x$.
Hence, both piles now have the same number of heads up. (shown)

2. The only possible set of values is $\{x = 2, y = 3, z = 6\}$.
Proofs
If $x = 2$ and $y \geqslant 4$, then $z \geqslant 5$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} < 1$.
If $x \geqslant 3$, then $y, z > 3$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} < 1$.

Chapter 6 Linear Equations

TEACHING NOTES

Suggested Approach

Since many Grade 6 students are still in the concrete operational stage (according to Piaget), teaching students how to solve linear equations in one variable with the use of algebra discs on a balance can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use this approach in examinations, and partly because they cannot use this approach to solve linear equations which consist of algebraic terms that have large coefficients (see Section 6.1). After students learn how to solve linear equations, they will learn how to evaluate an unknown in a formula and formulate linear equations to solve problems in real-world contexts.

Section 6.1: Linear Equations

Students have learnt how to complete mathematical sentences such as $7 + \square = 13$ in primary school. Teachers can introduce equations by telling students that when we replace $\square$ with x, we have $7 + x = 13$, which is an equation. Teachers should illustrate the meaning of 'solving an equation' using appropriate examples. Students should know the difference between linear expressions and linear equations.

Teachers can use the 'Balance Method' to show how to solve linear equations which do not involve any brackets before illustrating how to solve those which involve brackets. As this approach cannot be used to solve linear equations which consist of algebraic terms that have large or fractional coefficients, so there is a need to help students consolidate what they have learnt in Worked Examples 1, 2 and 3. The thinking time on page 107 of the textbook reinforces students' understanding of the concept of equation. For example, since $x + 3 = 6$, $2x + 3 = 9$ and $10x - 4 = 5x + 11$ are equivalent equations that can be obtained from $x = 3$, then the value of x in each of the equations is 3.

Section 6.2: Formulae

Teachers can use simple formulae such as $A = lb$, where A, l and b are the area, the length and the breadth of the rectangle respectively, to let students understand that a formula makes use of variables to write instructions for performing a calculation. Teachers may get students to provide examples of formulae which they have encountered in mathematics and the sciences.

Section 6.3: Applications of Linear Equations in Real-World Contexts

Teachers should illustrate how a word problem is solved using the model method before showing how the same problem can be solved using the algebraic method. Students should observe how the algebraic method is linked to the model method. Also, students should be aware why they need to learn the algebraic method. In this section, students are given ample opportunities to formulate linear equations to solve problems in real-world contexts.

WORKED SOLUTIONS

Journal Writing (Page 105)

1. To solve an equation in x means to find the value of x so that the values on both sides of the equation are equal, i.e. x satisfies the equation.
2. The operations should be applied to both sides of the equation such that the equation is simplified to the form $ax = b$, where a and b are constants. Thus $x = \frac{b}{a}$.

   Teachers may wish to point out common mistakes that students may make in solving a linear equation in order to extract their understanding of the process.

Thinking Time (Page 107)

Some equivalent equations that have the solution $y = -1$:

- $y = -1$
- $y + 1 = 0$
- $y - 1 = -2$
- $3y + 8 = 5$
- $2y - 1 = -3$
- $2y - 1 = -3$
- $10y + 2 = 13y + 5$
- $2(2y - 3) = 5(y - 1)$

Practise Now 1

(a)
$$\begin{aligned} x + 3 &= 7 \\ x + 3 - 3 &= 7 - 3 \\ \therefore x &= 4 \end{aligned}$$

(b)
$$\begin{aligned} x - 7 &= 6 \\ x - 7 + 7 &= 6 + 7 \\ \therefore x &= 13 \end{aligned}$$

(c)
$$\begin{aligned} x + 3 &= -7 \\ x + 3 - 3 &= -7 - 3 \\ \therefore x &= -10 \end{aligned}$$

(d)
$$\begin{aligned} x - 2 &= -3 \\ x - 2 + 2 &= -3 + 2 \\ \therefore x &= -1 \end{aligned}$$

Practise Now 2

(a)
$$\begin{aligned} 2x - 5 &= 5 \\ 2x - 5 + 5 &= 5 + 5 \\ 2x &= 10 \\ \therefore x &= 5 \end{aligned}$$

(b)
$$\begin{aligned} 3x + 4 &= 7 \\ 3x + 4 - 4 &= 7 - 4 \\ 3x &= 3 \\ \therefore x &= 1 \end{aligned}$$

(c)
$$\begin{aligned} -3x + 3 &= 9 \\ -3x + 3 - 3 &= 9 - 3 \\ -3x &= 6 \\ 3x &= -6 \\ \therefore x &= -2 \end{aligned}$$

(d)
$$\begin{aligned} -5x - 2 &= 13 \\ -5x - 2 + 2 &= 13 + 2 \\ -5x &= 15 \\ 5x &= -15 \\ \therefore x &= -3 \end{aligned}$$

Practise Now 3

(a)
$$\begin{aligned} 3x + 4 &= x - 10 \\ 3x - x + 4 &= x - x - 10 \\ 2x + 4 &= -10 \\ 2x + 4 - 4 &= -10 - 4 \\ 2x &= -14 \\ \therefore x &= -7 \end{aligned}$$

(b)
$$\begin{aligned} 4x - 2 &= x + 7 \\ 4x - x - 2 &= x - x + 7 \\ 3x - 2 &= 7 \\ 3x - 2 + 2 &= 7 + 2 \\ 3x &= 9 \\ \therefore x &= 3 \end{aligned}$$

(c)
$$\begin{aligned} 3x - 2 &= -x + 14 \\ 3x + x - 2 &= -x + x + 14 \\ 4x - 2 &= 14 \\ 4x - 2 + 2 &= 14 + 2 \\ 4x &= 16 \\ \therefore x &= 4 \end{aligned}$$

(d)
$$\begin{aligned} -2x - 5 &= 5x - 12 \\ -2x - 5x - 5 &= 5x - 5x - 12 \\ -7x - 5 &= -12 \\ -7x - 5 + 5 &= -12 + 5 \\ -7x &= -7 \\ 7x &= 7 \\ x &= 1 \end{aligned}$$

Practise Now 4

(a)
$$\begin{aligned} 2(x - 3) &= -3x + 4 \\ 2x - 6 &= -3x + 4 \\ 2x + 3x - 6 &= -3x + 3x + 4 \\ 5x - 6 &= 4 \\ 5x - 6 + 6 &= 4 + 6 \\ 5x &= 10 \\ \therefore x &= 2 \end{aligned}$$

(b)
$$\begin{aligned} 2(x + 3) &= 5x - 9 \\ 2x + 6 &= 5x - 9 \\ 2x - 5x + 6 &= 5x - 5x - 9 \\ -3x + 6 &= -9 \\ -3x + 6 - 6 &= -9 - 6 \\ -3x &= -15 \\ 3x &= 15 \\ \therefore x &= 5 \end{aligned}$$

(c) $-2(x + 2) = 3x - 9$

$-2x - 4 = 3x - 9$

$-2x - 3x - 4 = 3x - 3x - 9$

$-5x - 4 = -9$

$-5x - 4 + 4 = -9 + 4$

$-5x = -5$

$5x = 5$

$\therefore x = 1$

(d) $-2(3x - 4) = 4(2x + 5)$

$-6x + 8 = 8x + 20$

$-6x - 8x + 8 = 8x - 8x + 20$

$-14x + 8 = 20$

$-14x + 8 - 8 = 20 - 8$

$-14x = 12$

$14x = -12$

$\therefore x = -\frac{6}{7}$

Practise Now 5

1. (a) $x + 9 = 4$

$x + 9 - 9 = 4 - 9$

$\therefore x = -5$

(b) $3x - 2 = 4$

$3x - 2 + 2 = 4 + 2$

$3x = 6$

$\frac{3x}{3} = \frac{6}{3}$

$\therefore x = 2$

(c) $7x + 2 = 2x - 13$

$7x - 2x + 2 = 2x - 2x - 13$

$5x + 2 = -13$

$5x + 2 - 2 = -13 - 2$

$5x = -15$

$\frac{5x}{5} = \frac{-15}{5}$

$\therefore x = -3$

(d) $3(3y + 4) = 2(2y + 1)$

$9y + 12 = 4y + 2$

$9y - 4y + 12 = 4y - 4y + 2$

$5y + 12 = 2$

$5y + 12 - 12 = 2 - 12$

$5y = -10$

$\frac{5y}{5} = \frac{-10}{5}$

$\therefore y = -2$

(e) $2(y - 1) + 3(y - 1) = 4 - 2y$

$2y - 2 + 3y - 3 = 4 - 2y$

$2y + 3y - 2 - 3 = 4 - 2y$

$5y - 5 = 4 - 2y$

$5y + 2y - 5 = 4 - 2y + 2y$

$7y - 5 = 4$

$7y - 5 + 5 = 4 + 5$

$7y = 9$

$\frac{7y}{7} = \frac{9}{7}$

$\therefore y = 1\frac{2}{7}$

2. (a) $x + 0.7 = 2.7$

$x + 0.7 - 0.7 = 2.7 - 0.7$

$\therefore x = 2$

(b) $2y - 1.3 = 2.8$

$2y - 1.3 + 1.3 = 2.8 + 1.3$

$2y = 4.1$

$\frac{2y}{2} = \frac{4.1}{2}$

$\therefore y = 2.05$

Practise Now 6

(a) $\frac{x}{2} + 9 = 5$

$\frac{x}{2} + 9 - 9 = 5 - 9$

$\frac{x}{2} = -4$

$2 \times \frac{x}{2} = 2 \times (-4)$

$\therefore x = -8$

(b) $\frac{5}{7}y = 3\frac{1}{4} - 2$

$\frac{5}{7}y = \frac{13}{4} - 2$

$\frac{5}{7}y = \frac{13 - 8}{4}$

$\frac{5}{7}y = \frac{5}{4}$

$y = \frac{5}{4} \times \frac{7}{5}$

$y = \frac{7}{4}$

$\therefore y = 1\frac{3}{4}$

(c) $\dfrac{3z-1}{2} = \dfrac{z-4}{3}$

$6 \times \dfrac{3z-1}{2} = 6 \times \dfrac{z-4}{3}$

$3(3z-1) = 2(z-4)$

$9z - 3 = 2z - 8$

$9z - 2z - 3 = 2z - 2z - 8$

$7z - 3 = -8$

$7z - 3 + 3 = -8 + 3$

$7z = -5$

$\dfrac{7z}{7} = -\dfrac{5}{7}$

$\therefore z = -\dfrac{5}{7}$

Practise Now 7

(a) $\dfrac{0.5x-3}{8} = \dfrac{1}{4}$

$0.5x - 3 = \dfrac{1}{4} \times 8$

$0.5x - 3 = 2$

$0.5x = 2 + 3$

$0.5x = 5$

$x = \dfrac{5}{0.5}$

$\therefore x = 10$

(b) $\dfrac{6.5x}{0.5} = 13$

$13x = 13$

$x = \dfrac{13}{13}$

$\therefore x = 1$

Practise Now 8

$F = ma$

(a) When $m = 1000$, $a = 0.05$,

$F = 1000(0.05)$

$= 50$ N

Net force acting on body = 50 N

(b) When $F = 100$, $a = 0.1$,

$100 = m(0.1)$

$\therefore m = \dfrac{100}{0.1}$

$= 1000$ kg

Mass of body = 1000 kg

Practise Now 9

1. $\dfrac{2x+y-3z}{y+3x} = \dfrac{x}{2y}$

When $x = 1$, $y = 4$,

$\dfrac{2(1)+4-3z}{4+3(1)} = \dfrac{1}{2(4)}$

$\dfrac{2+4-3z}{4+3} = \dfrac{1}{8}$

$\dfrac{6-3z}{7} = \dfrac{1}{8}$

$8(6-3z) = 7$

$48 - 24z = 7$

$-24z = 7 - 48$

$-24z = -41$

$\therefore z = \dfrac{-41}{-24}$

$= 1\dfrac{17}{24}$

2. $t = \dfrac{v-u}{a}$

When $t = 3$, $v = 2\dfrac{1}{2}$, $u = 1\dfrac{1}{3}$,

$3 = \dfrac{2\frac{1}{2} - 1\frac{1}{3}}{a}$

$3 = \dfrac{1\frac{1}{6}}{a}$

$3a = 1\dfrac{1}{6}$

$\therefore a = 1\dfrac{1}{6} \div 3$

$= \dfrac{7}{18}$

Practise Now 10

(i) $A = \dfrac{1}{2}\pi r^2$

(ii) When $r = 5$,

$A = \dfrac{1}{2}(3.142)(5)^2$

$= 39.275$ cm^2

Area of semicircle = 39.275 cm^2

Practise Now 11

1. Let the smaller number be x.

Then the larger number is $5x$.

$x + 5x = 24$

$6x = 24$

$x = \dfrac{24}{6}$

$= 4$

$\therefore$ The two numbers are 4 and 20.

2. Let the number of marks Lixin obtains be x.

Then the number of marks Devi obtains is $x + 15$.

$$x + 15 = 2x$$
$$x - 2x = -15$$
$$-x = -15$$
$$\therefore x = 15$$

Lixin obtains 15 marks.

Practise Now 12

Let the number be x.

$$\frac{1}{5}x + 3\frac{7}{10} = 7$$
$$\frac{1}{5}x = 7 - 3\frac{7}{10}$$
$$\frac{1}{5}x = 3\frac{3}{10}$$
$$\therefore x = 5 \times 3\frac{3}{10}$$
$$= 16\frac{1}{2}$$

The number is $16\frac{1}{2}$.

Exercise 6A

1. **(a)** $x + 8 = 15$

$x + 8 - 8 = 15 - 8$

$\therefore x = 7$

(b) $x + 9 = -5$

$x + 9 - 9 = -5 - 9$

$\therefore x = -14$

(c) $x - 5 = 17$

$x - 5 + 5 = 17 + 5$

$\therefore x = 22$

(d) $y - 7 = -3$

$y - 7 + 7 = -3 + 7$

$\therefore y = 4$

(e) $y + 0.4 = 1.6$

$y + 0.4 - 0.4 = 1.6 - 0.4$

$\therefore y = 1.2$

(f) $y - 2.4 = 3.6$

$y - 2.4 + 2.4 = 3.6 + 2.4$

$y = 6$

(g) $-2.7 + a = -6.4$

$-2.7 + 2.7 + a = -6.4 + 2.7$

$\therefore a = -3.7$

2. **(a)** $4x = -28$

$$\frac{4x}{4} = \frac{-28}{4}$$

$\therefore x = -7$

(b) $-24x = -144$

$24x = 144$

$$\frac{24x}{24} = \frac{144}{24}$$

$\therefore x = 6$

(c) $3x - 4 = 11$

$3x - 4 + 4 = 11 + 4$

$3x = 15$

$$\frac{3x}{3} = \frac{15}{3}$$

$\therefore x = 5$

(d) $9x + 4 = 31$

$9x + 4 - 4 = 31 - 4$

$9x = 27$

$$\frac{9x}{9} = \frac{27}{9}$$

$\therefore x = 3$

(e) $12 - 7x = 5$

$12 - 12 - 7x = 5 - 12$

$-7x = -7$

$7x = 7$

$$\frac{7x}{7} = \frac{7}{7}$$

$\therefore x = 1$

(f) $3 - 7y = -18$

$3 - 3 - 7y = -18 - 3$

$-7y = -21$

$7y = 21$

$$\frac{7y}{7} = \frac{21}{7}$$

$\therefore y = 3$

(g) $4y - 1.9 = 6.3$

$4y - 1.9 + 1.9 = 6.3 + 1.9$

$4y = 8.2$

$$\frac{4y}{4} = \frac{8.2}{4}$$

$\therefore y = 2.05$

(h) $-3y - 7.8 = -9.6$

$-3y - 7.8 + 7.8 = -9.6 + 7.8$

$-3y = -1.8$

$3y = 1.8$

$$\frac{3y}{3} = \frac{1.8}{3}$$

$\therefore y = 0.6$

(i) $7y - 2\frac{3}{4} = \frac{1}{2}$

$$7y - 2\frac{3}{4} + 2\frac{3}{4} = \frac{1}{2} + 2\frac{3}{4}$$

$$7y = 3\frac{1}{4}$$

$$\frac{7y}{7} = 3\frac{1}{4} \div 7$$

$$\therefore y = \frac{13}{28}$$

(j) $1\frac{1}{2} - 2y = \frac{1}{4}$

$1\frac{1}{2} - 1\frac{1}{2} - 2y = \frac{1}{4} - 1\frac{1}{2}$

$-2y = -1\frac{1}{4}$

$2y = 1\frac{1}{4}$

$\frac{2y}{2} = 1\frac{1}{4} \div 2$

$\therefore y = \frac{5}{8}$

3. **(a)** $3x - 7 = 4 - 8x$

$3x + 8x - 7 = 4 - 8x + 8x$

$11x - 7 = 4$

$11x - 7 + 7 = 4 + 7$

$11x = 11$

$\frac{11x}{11} = \frac{11}{11}$

$\therefore x = 1$

(b) $4x - 10 = 5x + 7$

$4x - 5x - 10 = 5x - 5x + 7$

$-x - 10 = 7$

$-x - 10 + 10 = 7 + 10$

$-x = 17$

$\therefore x = -17$

(c) $30 + 7y = -2y - 6$

$30 + 7y + 2y = -2y + 2y - 6$

$30 + 9y = -6$

$30 - 30 + 9y = -6 - 30$

$9y = -36$

$\frac{9y}{9} = \frac{-36}{9}$

$\therefore y = -4$

(d) $2y - 7 = 7y - 27$

$2y - 7y - 7 = 7y - 7y - 27$

$-5y - 7 = -27$

$-5y - 7 + 7 = -27 + 7$

$-5y = -20$

$5y = 20$

$\frac{5y}{5} = \frac{20}{5}$

$\therefore y = 4$

4. **(a)** $2(x + 3) = 8$

$2x + 6 = 8$

$2x + 6 - 6 = 8 - 6$

$2x = 2$

$\frac{2x}{2} = \frac{2}{2}$

$\therefore x = 1$

(b) $5(x - 7) = -15$

$5x - 35 = -15$

$5x - 35 + 35 = -15 + 35$

$5x = 20$

$\frac{5x}{5} = \frac{20}{5}$

$\therefore x = 4$

(c) $7(-2x + 4) = -4x$

$-14x + 28 = -4x$

$-14x + 4x + 28 = -4x + 4x$

$-10x + 28 = 0$

$-10x + 28 - 28 = 0 - 28$

$-10x = -28$

$10x = 28$

$\frac{10x}{10} = \frac{28}{10}$

$\therefore x = 2\frac{4}{5}$

(d) $3(2 - 0.4x) = 18$

$6 - 1.2x = 18$

$6 - 6 - 1.2x = 18 - 6$

$-1.2x = 12$

$1.2x = -12$

$\frac{1.2x}{1.2} = \frac{-12}{1.2}$

$\therefore x = -10$

(e) $2(2x - 2.2) = 4.6$

$4x - 4.4 = 4.6$

$4x - 4.4 + 4.4 = 4.6 + 4.4$

$4x = 9$

$\frac{4x}{4} = \frac{9}{4}$

$\therefore x = 2.25$

(f) $4(3y + 4.1) = 7.6$

$12y + 16.4 = 7.6$

$12y + 16.4 - 16.4 = 7.6 - 16.4$

$12y = -8.8$

$\frac{12y}{12} = \frac{-8.8}{12}$

$\therefore y = -\frac{11}{15}$

(g) $3(2y + 3) = 4y + 3$

$6y + 9 = 4y + 3$

$6y - 4y + 9 = 4y - 4y + 3$

$2y + 9 = 3$

$2y + 9 - 9 = 3 - 9$

$2y = -6$

$\frac{2y}{2} = \frac{-6}{2}$

$\therefore y = -3$

(h) $3(y + 1) = 4y - 21$

$3y + 3 = 4y - 21$

$3y - 4y + 3 = 4y - 4y - 21$

$-y + 3 = -21$

$-y + 3 - 3 = -21 - 3$

$-y = -24$

$\therefore y = 24$

(i) $3(y + 2) = 2(y + 4)$

$3y + 6 = 2y + 8$

$3y - 2y + 6 = 2y - 2y + 8$

$y + 6 = 8$

$y + 6 - 6 = 8 - 6$

$\therefore y = 2$

(j) $5(5y - 6) = 4(y - 7)$

$25y - 30 = 4y - 28$

$25y - 4y - 30 = 4y - 4y - 28$

$21y - 30 = -28$

$21y - 30 + 30 = -28 + 30$

$21y = 2$

$\frac{21y}{21} = \frac{2}{21}$

$\therefore y = \frac{2}{21}$

(k) $2(3b - 4) = 5(b + 6)$

$6b - 8 = 5b + 30$

$6b - 5b - 8 = 5b - 5b + 30$

$b - 8 = 30$

$b - 8 + 8 = 30 + 8$

$\therefore b = 38$

(l) $3(2c + 5) = 4(c - 3)$

$6c + 15 = 4c - 12$

$6c - 4c + 15 = 4c - 4c - 12$

$2c + 15 = -12$

$2c + 15 - 15 = -12 - 15$

$2c = -27$

$\frac{2c}{2} = \frac{-27}{2}$

$\therefore c = -13\frac{1}{2}$

(m) $9(2d + 7) = 11(d + 14)$

$18d + 63 = 11d + 154$

$18d - 11d + 63 = 11d - 11d + 154$

$7d + 63 = 154$

$7d + 63 - 63 = 154 - 63$

$7d = 91$

$\frac{7d}{7} = \frac{91}{7}$

$\therefore d = 13$

(n) $5(7f - 3) = 28(f - 1)$

$35f - 15 = 28f - 28$

$35f - 28f - 15 = 28f - 28f - 28$

$7f - 15 = -28$

$7f - 15 + 15 = -28 + 15$

$7f = -13$

$\frac{7f}{7} = \frac{-13}{7}$

$\therefore f = -1\frac{6}{7}$

5. **(a)** $\frac{1}{3}x = 7$

$3 \times \frac{1}{3}x = 3 \times 7$

$\therefore x = 21$

(b) $\frac{3}{4}x = -6$

$\frac{4}{3} \times \frac{3}{4}x = \frac{4}{3} \times (-6)$

$\therefore x = -8$

(c) $\frac{1}{3}x + 3 = 4$

$\frac{1}{3}x + 3 - 3 = 4 - 3$

$\frac{1}{3}x = 1$

$3 \times \frac{1}{3}x = 3 \times 1$

$\therefore x = 3$

(d) $\frac{y}{4} - 8 = -2$

$\frac{y}{4} - 8 + 8 = -2 + 8$

$\frac{y}{4} = 6$

$4 \times \frac{y}{4} = 4 \times 6$

$\therefore y = 24$

(e) $3 - \frac{1}{4}y = 2$

$3 - 3 - \frac{1}{4}y = 2 - 3$

$-\frac{1}{4}y = -1$

$\frac{1}{4}y = 1$

$4 \times \frac{1}{4}y = 4 \times 1$

$\therefore y = 4$

(f) $15 - \frac{2}{5}y = 11$

$15 - 15 - \frac{2}{5}y = 11 - 15$

$-\frac{2}{5}y = -4$

$\frac{2}{5}y = 4$

$\frac{5}{2} x \frac{2}{5}y = \frac{5}{2} \times 4$

$\therefore y = 10$

6. **(a)** $x = 12 - \frac{1}{3}x$

$x + \frac{1}{3}x = 12 - \frac{1}{3}x + \frac{1}{3}x$

$\frac{4}{3}x = 12$

$\frac{3}{4} \times \frac{4}{3}x = \frac{3}{4} \times 12$

$\therefore x = 9$

(b) $\frac{3}{5}x = \frac{1}{2}x + \frac{1}{2}$

$\frac{3}{5}x - \frac{1}{2}x = \frac{1}{2}x - \frac{1}{2}x + \frac{1}{2}$

$\frac{1}{10}x = \frac{1}{2}$

$10 \times \frac{1}{10}x = 10 \times \frac{1}{2}$

$\therefore x = 5$

(c) $\frac{y}{2} - \frac{1}{5} = 2 - \frac{y}{3}$

$\frac{y}{2} + \frac{y}{3} - \frac{1}{5} = 2 - \frac{y}{3} + \frac{y}{3}$

$\frac{5y}{6} - \frac{1}{5} = 2$

$\frac{5y}{6} - \frac{1}{5} + \frac{1}{5} = 2 + \frac{1}{5}$

$\frac{5y}{6} = 2\frac{1}{5}$

$\frac{6}{5} \times \frac{5y}{6} = \frac{6}{5} \times 2\frac{1}{5}$

$\therefore y = 2\frac{16}{25}$

(d) $\frac{2}{3}y - \frac{3}{4} = 2y + \frac{5}{8}$

$\frac{2}{3}y - 2y - \frac{3}{4} = 2y - 2y + \frac{5}{8}$

$-\frac{4}{3}y - \frac{3}{4} = \frac{5}{8}$

$-\frac{4}{3}y - \frac{3}{4} + \frac{3}{4} = \frac{5}{8} + \frac{3}{4}$

$-\frac{4}{3}y = 1\frac{3}{8}$

$\frac{4}{3}y = -1\frac{3}{8}$

$\frac{3}{4} \times \frac{4}{3}y = \frac{3}{4} \times -1\frac{3}{8}$

$\therefore y = -1\frac{1}{32}$

7. **(a)** $\frac{0.2}{x} = 0.8$

$x \times \frac{0.2}{x} = 0.8 \times x$

$0.2 = 0.8x$

$\frac{0.8x}{0.8} = \frac{0.2}{0.8}$

$= 0.25$

(b) $\frac{1.2}{y-1} = 0.6$

$(y-1) \times \frac{1.2}{y-1} = 0.6 \times (y-1)$

$1.2 + 0.6 = 0.6y - 0.6 + 0.6$

$1.8 = 0.6y$

$\frac{1.8}{0.6} = \frac{0.6y}{0.6}$

$y = 3$

8. **(a)** $-3(2 - x) = 6x$

$-6 + 3x = 6x$

$-6 + 3x - 6x = 6x - 6x$

$-6 - 3x = 0$

$-6 + 6 - 3x = 0 + 6$

$-3x = 6$

$3x = -6$

$\frac{3x}{3} = \frac{-6}{3}$

$\therefore x = -2$

(b) $5 - 3x = -6(x + 2)$

$5 - 3x = -6x - 12$

$5 - 3x + 6x = -6x + 6x - 12$

$5 + 3x = -12$

$5 - 5 + 3x = -12 - 5$

$3x = -17$

$\frac{3x}{3} = \frac{-17}{3}$

$\therefore x = -5\frac{2}{3}$

(c) $-3(9y + 2) = 2(-4y - 7)$

$$-27y - 6 = -8y - 14$$
$$-27y + 8y - 6 = -8y + 8y - 14$$
$$-19y - 6 = -14$$
$$-19y - 6 + 6 = -14 + 6$$
$$-19y = -8$$
$$19y = 8$$
$$\frac{19y}{19} = \frac{8}{19}$$
$$\therefore y = \frac{8}{19}$$

(d) $-3(4y - 5) = -7(-5 - 2y)$

$$-12y + 15 = 35 + 14y$$
$$-12y - 14y + 15 = 35 + 14y - 14y$$
$$-26y + 15 = 35$$
$$-26y + 15 - 15 = 35 - 15$$
$$-26y = 20$$
$$26y = -20$$
$$\frac{26y}{26} = \frac{-20}{26}$$
$$\therefore y = -\frac{10}{13}$$

(e) $3(5 - h) - 2(h - 2) = -1$

$$15 - 3h - 2h + 4 = -1$$
$$15 + 4 - 3h - 2h = -1$$
$$19 - 5h = -1$$
$$19 - 19 - 5h = -1 - 19$$
$$-5h = -20$$
$$5h = 20$$
$$\frac{5h}{5} = \frac{20}{5}$$
$$\therefore h = 4$$

9. (a) $\frac{5x + 1}{3} = 7$

$$3 \times \frac{5x + 1}{3} = 3 \times 7$$
$$5x + 1 = 21$$
$$5x + 1 - 1 = 21 - 1$$
$$5x = 20$$
$$\frac{5x}{5} = \frac{20}{5}$$
$$\therefore x = 4$$

(b) $\frac{2x - 3}{4} = \frac{x - 3}{3}$

$$12 \times \frac{2x - 3}{4} = 12 \times \frac{x - 3}{3}$$
$$3(2x - 3) = 4(x - 3)$$
$$6x - 9 = 4x - 12$$
$$6x - 4x - 9 = 4x - 4x - 12$$
$$2x - 9 = -12$$
$$2x - 9 + 9 = -12 + 9$$
$$2x = -3$$
$$\frac{2x}{2} = \frac{-3}{2}$$
$$\therefore x = -1\frac{1}{2}$$

(c) $\frac{3x - 1}{5} = \frac{x - 1}{3}$

$$15 \times \frac{3x - 1}{5} = 15 \times \frac{x - 1}{3}$$
$$3(3x - 1) = 5(x - 1)$$
$$9x - 3 = 5x - 5$$
$$9x - 5x - 3 = 5x - 5x - 5$$
$$4x - 3 = -5$$
$$4x - 3 + 3 = -5 + 3$$
$$4x = -2$$
$$\frac{4x}{4} = \frac{-2}{4}$$
$$\therefore x = -\frac{1}{2}$$

(d) $\frac{1}{4}(5y + 4) = \frac{1}{3}(2y - 1)$

$$12 \times \frac{1}{4}(5y + 4) = 12 \times \frac{1}{3}(2y - 1)$$
$$3(5y + 4) = 4(2y - 1)$$
$$15y + 12 = 8y - 4$$
$$15y - 8y + 12 = 8y - 8y - 4$$
$$7y + 12 = -4$$
$$7y + 12 - 12 = -4 - 12$$
$$7y = -16$$
$$\frac{7y}{7} = \frac{-16}{7}$$
$$\therefore y = -2\frac{2}{7}$$

(e) $\frac{2y - 1}{5} - \frac{y + 3}{7} = 0$

$$\frac{2y - 1}{5} - \frac{y + 3}{7} + \frac{y + 3}{7} = 0 + \frac{y + 3}{7}$$
$$\frac{2y - 1}{5} = \frac{y + 3}{7}$$
$$35 \times \frac{2y - 1}{5} = 35 \times \frac{y + 3}{7}$$
$$7(2y - 1) = 5(y + 3)$$
$$14y - 7 = 5y + 15$$
$$14y - 5y - 7 = 5y - 5y + 15$$
$$9y - 7 = 15$$
$$9y - 7 + 7 = 15 + 7$$
$$9y = 22$$
$$\frac{9y}{9} = \frac{22}{9}$$
$$\therefore y = 2\frac{4}{9}$$

(f) $$\frac{2y+3}{4} + \frac{y-5}{6} = 0$$

$$\frac{2y+3}{4} + \frac{y-5}{6} - \frac{y-5}{6} = 0 - \frac{5-y}{6}$$

$$\frac{2y+3}{4} = -\frac{5-y}{6}$$

$$\frac{2y+3}{4} = \frac{y-5}{6}$$

$$12 \times \frac{2y+3}{4} = 12 \times \frac{y-5}{6}$$

$$3(2y+3) = 2(5-y)$$

$$6y + 9 = 10 - 2y$$

$$6y + 2y + 9 = 10 - 2y + 2y$$

$$8y + 9 = 10$$

$$8y + 9 - 9 = 10 - 9$$

$$8y = 1$$

$$\frac{8y}{8} = \frac{1}{8}$$

$$\therefore y = \frac{1}{8}$$

10. (a) $$\frac{12}{x+3} = 2$$

$$(x+3) \times \frac{12}{x+3} = (x+3) \times 2$$

$$12 = 2(x+3)$$

$$12 = 2x + 6$$

$$2x + 6 = 12$$

$$2x + 6 - 6 = 12 - 6$$

$$2x = 6$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$\therefore x = 3$$

(b) $$\frac{11}{2x-1} = 4$$

$$(2x-1) \times \frac{11}{2x-1} = (2x-1) \times 4$$

$$11 = 4(2x-1)$$

$$11 = 8x - 4$$

$$8x - 4 = 11$$

$$8x - 4 + 4 = 11 + 4$$

$$8x = 15$$

$$\frac{8x}{8} = \frac{15}{8}$$

$$\therefore x = 1\frac{7}{8}$$

(c) $$\frac{32}{2x-5} - 3 = \frac{1}{4}$$

$$\frac{32}{2x-5} - 3 + 3 = \frac{1}{4} + 3$$

$$\frac{32}{2x-5} = \frac{13}{4}$$

$$4(2x-5) \times \frac{32}{2x-5} = 4(2x-5) \times \frac{13}{4}$$

$$128 = 13(2x-5)$$

$$128 = 26x - 65$$

$$26x - 65 = 128$$

$$26x - 65 + 65 = 128 + 65$$

$$26x = 193$$

$$\frac{26x}{26} = \frac{193}{26}$$

$$\therefore x = 7\frac{11}{26}$$

(d) $$\frac{1}{2} = \frac{1}{x+2} - 1$$

$$\frac{1}{x+2} - 1 = \frac{1}{2}$$

$$\frac{1}{x+2} - 1 + 1 = \frac{1}{2} + 1$$

$$\frac{1}{x+2} = \frac{3}{2}$$

$$2(x+2) \times \frac{1}{x+2} = 2(x+2) \times \frac{3}{2}$$

$$2 = 3(x+2)$$

$$2 = 3x + 6$$

$$3x + 6 = 2$$

$$3x + 6 - 6 = 2 - 6$$

$$3x = -4$$

$$\frac{3x}{3} = \frac{-4}{3}$$

$$\therefore x = -1\frac{1}{3}$$

(e) $$\frac{y+5}{y-6} = \frac{5}{4}$$

$$4(y-6) \times \frac{y+5}{y-6} = 4(y-6) \times \frac{5}{4}$$

$$4(y+5) = 5(y-6)$$

$$4y + 20 = 5y - 30$$

$$4y - 5y + 20 = 5y - 5y - 30$$

$$-y + 20 = -30$$

$$-y + 20 - 20 = -30 - 20$$

$$-y = -50$$

$$\therefore y = 50$$

(f) $\frac{2y+1}{3y-5} = \frac{4}{7}$

$7(3y-5) \times \frac{2y+1}{3y-5} = 7(3y-5) \times \frac{4}{7}$

$7(2y+1) = 4(3y-5)$

$14y + 7 = 12y - 20$

$14y - 12y + 7 = 12y - 12y - 20$

$2y + 7 = -20$

$2y + 7 - 7 = -20 - 7$

$2y = -27$

$\frac{2y}{2} = \frac{-27}{2}$

$\therefore y = -13\frac{1}{2}$

(g) $\frac{2}{y-2} = \frac{3}{y+6}$

$(y-2)(y+6) \times \frac{2}{y-2} = (y-2)(y+6) \times \frac{3}{y+6}$

$2(y+6) = 3(y-2)$

$2y + 12 = 3y - 6$

$2y - 3y + 12 = 3y - 3y - 6$

$-y + 12 = -6$

$-y + 12 - 12 = -6 - 12$

$-y = -18$

$\therefore y = 18$

(h) $\frac{2}{7y-3} = \frac{3}{9y-5}$

$(7y-3)(9y-5) \times \frac{2}{7y-3} = (7y-3)(9y-5) \times \frac{3}{9y-5}$

$2(9y-5) = 3(7y-3)$

$18y - 10 = 21y - 9$

$18y - 21y - 10 = 21y - 21y - 9$

$-3y - 10 = -9$

$-3y - 10 + 10 = -9 + 10$

$-3y = 1$

$3y = 1$

$\frac{3y}{3} = \frac{-1}{3}$

$\therefore y = -\frac{1}{3}$

11. (a) $10x - \frac{5x+4}{3} = 7$

$\frac{3(10x) - (5x+4)}{3} = 7$

$\frac{30x - 5x - 4}{3} = 7$

$\frac{25x - 4}{3} = 7$

$3 \times \frac{25x-4}{3} = 3 \times 7$

$25x - 4 = 21$

$25x - 4 + 4 = 21 + 4$

$25x = 25$

$\frac{25x}{25} = \frac{25}{25}$

$\therefore x = 1$

(b) $\frac{4x}{3} - \frac{x-1}{2} = 1\frac{1}{4}$

$\frac{2(4x) - 3(x-1)}{6} = \frac{5}{4}$

$\frac{8x - 3x + 3}{6} = \frac{5}{4}$

$\frac{5x+3}{6} = \frac{5}{4}$

$12 \times \frac{5x+3}{6} = 12 \times \frac{5}{4}$

$2(5x+3) = 15$

$10x + 6 = 15$

$10x + 6 - 6 = 15 - 6$

$10x = 9$

$\frac{10x}{10} = \frac{9}{10}$

$\therefore x = \frac{9}{10}$

(c) $\frac{x-1}{3} - \frac{x+3}{4} = -1$

$\frac{4(x-1) - 3(x+3)}{12} = -1$

$\frac{4x - 4 - 3x - 9}{12} = -1$

$\frac{4x - 3x - 4 - 9}{12} = -1$

$\frac{x - 13}{12} = -1$

$12 \times \frac{x-13}{12} = 12 \times (-1)$

$x - 13 = -12$

$x - 13 + 13 = -12 + 13$

$\therefore x = 1$

(d) $1 - \frac{y+5}{3} = \frac{3(y-1)}{4}$

$\frac{3 - (y+5)}{3} = \frac{3(y-1)}{4}$

$\frac{3 - y - 5}{3} = \frac{3(y-1)}{4}$

$\frac{-y + 3 - 5}{3} = \frac{3(y-1)}{4}$

$\frac{-y - 2}{3} = \frac{3(y-1)}{4}$

$12 \times \frac{-y-2}{3} = 12 \times \frac{3(y-1)}{4}$

$4(-y-2) = 9(y-1)$

$-4y - 8 = 9y - 9$

$-4y - 9y - 8 = 9y - 9y - 9$

$-13y - 8 = -9$

$-13y - 8 + 8 = -9 + 8$

$-13y = -1$

$13y = 1$

$\frac{13y}{13} = \frac{1}{13}$

$\therefore y = \frac{1}{13}$

(e) $\frac{6(y-2)}{7} - 12 = \frac{2(y-7)}{3}$

$$\frac{6(y-2)-84}{7} = \frac{2(y-7)}{3}$$

$$\frac{6y-12-84}{7} = \frac{2(y-7)}{3}$$

$$\frac{6y-96}{7} = \frac{2(y-7)}{3}$$

$$21 \times \frac{6y-96}{7} = 21 \times \frac{2(y-7)}{3}$$

$$3(6y-96) = 14(y-7)$$

$$18y - 288 = 14y - 98$$

$$18y - 14y - 288 = 14y - 14y - 98$$

$$4y - 288 = -98$$

$$4y - 288 + 288 = -98 + 288$$

$$4y = 190$$

$$\frac{4y}{4} = \frac{190}{4}$$

$$\therefore\ y = 47\frac{1}{2}$$

(f) $\frac{7-2y}{2} - \frac{2}{5}(2-y) = 1\frac{1}{4}$

$$\frac{5(7-2y)-4(2-y)}{10} = \frac{5}{4}$$

$$\frac{35-10y-8+4y}{10} = \frac{5}{4}$$

$$\frac{-10y+4y+35-8}{10} = \frac{5}{4}$$

$$\frac{-6y+27}{10} = \frac{5}{4}$$

$$20 \times \frac{-6y+27}{10} = 20 \times \frac{5}{4}$$

$$2(-6y+27) = 25$$

$$-12y + 54 = 25$$

$$-12y + 54 - 54 = 25 - 54$$

$$-12y = -29$$

$$12y = 29$$

$$\frac{12y}{12} = \frac{29}{12}$$

$$\therefore\ y = 2\frac{5}{12}$$

12. When $x = \frac{19}{20}$,

$$\text{LHS} = 2 \quad \frac{19}{20} \quad - \frac{3}{4}$$

$$= 1\frac{9}{10} - \frac{3}{4}$$

$$= 1\frac{3}{20}$$

$$\text{RHS} = \frac{1}{3} \quad \frac{19}{20} \quad + \frac{5}{6}$$

$$= \frac{19}{60} + \frac{5}{6}$$

$$= 1\frac{3}{20} = \text{LHS}$$

$\therefore\ x = \frac{19}{20}$ is the solution of the equation

$2x - \frac{3}{4} = \frac{1}{3}x + \frac{5}{6}$.

13. $4x + y = 3x + 5y$

$$4x - 3x + y = 3x - 3x + 5y$$

$$x + y = 5y$$

$$x + y - y = 5y - y$$

$$x = 4y$$

$$\frac{3}{16y} \times x = \frac{3}{16y} \times 4y$$

$$\therefore\ \frac{3x}{16y} = \frac{3}{4}$$

14. $\frac{3x-5y}{7x-4y} = \frac{3}{4}$

$$4(7x-4y) \times \frac{3x-5y}{7x-4y} = 4(7x-4y) \times \frac{3}{4}$$

$$4(3x-5y) = 3(7x-4y)$$

$$12x - 20y = 21x - 12y$$

$$12x - 21x - 20y = 21x - 21x - 12y$$

$$-9x - 20y = -12y$$

$$-9x - 20y + 20y = -12y + 20y$$

$$-9x = 8y$$

$$9x = -8y$$

$$\frac{9x}{9} = \frac{-8y}{9}$$

$$x = -\frac{8y}{9}$$

$$\frac{1}{y} \times x = \frac{1}{y} \times \quad -\frac{8y}{9}$$

$$\therefore\ \frac{x}{y} = -\frac{8}{9}$$

Exercise 6B

1. $y = \frac{3}{5}x + 26$

When $x = 12$,

$$y = \frac{3}{5}(12) + 26$$

$$= 33\frac{1}{5}$$

2. $a = \dfrac{y^2 - xz}{5}$

When $x = 2$, $y = -1$, $z = -3$,

$a = \dfrac{(-1)^2 - 2(-3)}{5}$

$= \dfrac{1 + 6}{5}$

$= \dfrac{7}{5}$

$= 1\dfrac{2}{5}$

3. $S = 4\pi r^2$

(i) When $r = 10\dfrac{1}{2}$,

$S = 4\ \dfrac{22}{7}\ \ 10\dfrac{1}{2}\ ^2$

$= 1386$

(ii) When $S = 616$,

$616 = 4\ \dfrac{22}{7}\ r^2$

$616 = \dfrac{88}{7}r^2$

$\dfrac{88}{7}r^2 = 616$

$r^2 = \dfrac{7}{88} \times 616$

$r^2 = 49$

$\therefore r = \pm\sqrt{49}$

$= \pm 7$

$= 7$ or -7 (N.A. since $r > 0$)

4. $A = \dfrac{1}{2}bh$

(i) When $b = 20$, $h = 45$,

$A = \dfrac{1}{2}(20)(45)$

$= 450\ \text{cm}^2$

Area of triangle = 450 cm^2

(ii) When $A = 30$, $b = 10$,

$30 = \dfrac{1}{2}(10)h$

$30 = 5h$

$5h = 30$

$\therefore h = 6$

Height of triangle = 6 cm

5. **(a)** $P = xyz$

(b) $S = p^2 + q^3$

(c) $A = \dfrac{m + n + p + q}{4}$

(d) $T = 60a + b$

6. $k = \dfrac{p + 2q}{3}$

When $k = 7$, $q = 9$,

$7 = \dfrac{p + 2(9)}{3}$

$7 = \dfrac{p + 18}{3}$

$3 \times 7 = p + 18$

$21 = p + 18$

$p + 18 = 21$

$\therefore p = 21 - 18$

$= 3$

7. $U = \pi(r + h)$

When $U = 16\dfrac{1}{2}$, $h = 2$,

$16\dfrac{1}{2} = \dfrac{22}{7}(r + 2)$

$\dfrac{22}{7}(r + 2) = 16\dfrac{1}{2}$

$r + 2 = \dfrac{7}{22} \times 16\dfrac{1}{2}$

$r + 2 = 5\dfrac{1}{4}$

$\therefore r = 5\dfrac{1}{4} - 2$

$= 3\dfrac{1}{4}$

8. $v^2 = u^2 + 2gs$

When $v = 25$, $u = 12$, $g = 10$,

$25^2 = 12^2 + 2(10)s$

$625 = 144 + 20s$

$144 + 20s = 625$

$20s = 625 - 144$

$20s = 481$

$\therefore s = \dfrac{481}{20}$

$= 24\dfrac{1}{20}$

9. $\dfrac{a}{b} - d = \dfrac{2c}{b}$

When $a = 3$, $b = 4$, $d = -5$,

$\dfrac{3}{4} - (-5) = \dfrac{2c}{4}$

$5\dfrac{3}{4} = \dfrac{c}{2}$

$\dfrac{c}{2} = 5\dfrac{3}{4}$

$\therefore c = 2 \times 5\dfrac{3}{4}$

$= 11\dfrac{1}{2}$

10. $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} + \frac{1}{d}$

When $a = \frac{1}{2}$, $b = \frac{1}{4}$, $d = -\frac{1}{5}$,

$$\frac{1}{\frac{1}{2}} - \frac{1}{\frac{1}{4}} = \frac{1}{c} + \frac{1}{\frac{-1}{5}}$$

$$2 - 4 = \frac{1}{c} - 5$$

$$-2 = \frac{1}{c} - 5$$

$$-2 + 5 = \frac{1}{c}$$

$$3 = \frac{1}{c}$$

$$3c = 1$$

$$\therefore c = \frac{1}{3}$$

11. $N = \frac{m}{x + q}$

When $N = 1\frac{4}{5}$, $m = 9$, $x = 2$,

$$1\frac{4}{5} = \frac{9}{2 + q}$$

$$\frac{9}{5} = \frac{9}{2 + q}$$

$$\frac{1}{5} = \frac{1}{2 + q}$$

$$2 + q = 5$$

$$\therefore q = 5 - 2$$

$$= 3$$

12. $c = \frac{a}{b} - \frac{d - e}{f - d}$

When $a = 3$, $b = 4$, $c = -6$, $d = -5$ and $e = 2$,

$$-6 = \frac{3}{4} - \frac{-5 - 2}{f - (-5)}$$

$$-6 = \frac{3}{4} - \frac{-7}{f + 5}$$

$$-6 = \frac{3}{4} + \frac{7}{f + 5}$$

$$-6 - \frac{3}{4} = \frac{7}{f + 5}$$

$$-6\frac{3}{4} = \frac{7}{f + 5}$$

$$-\frac{27}{4} = \frac{7}{f + 5}$$

$$-27(f + 5) = 4 \times 7$$

$$-27(f + 5) = 28$$

$$-27f - 135 = 28$$

$$-27f = 28 + 135$$

$$-27f = 163$$

$$\therefore f = \frac{163}{-27}$$

$$= -6\frac{1}{27}$$

13. $a = \frac{b}{c - b}$

When $a = 3$, $c = 10$,

$$3 = \frac{b}{10 - b}$$

$$3(10 - b) = b$$

$$30 - 3b = b$$

$$-3b - b = -30$$

$$-4b = -30$$

$$\therefore b = \frac{-30}{-4}$$

$$= 7\frac{1}{2}$$

14. $\frac{m(nx^2 - y)}{z} = 5n$

When $m = 6$, $x = -2$, $y = -3$, $z = -5$,

$$\frac{6[n(-2)^2 - (-3)]}{-5} = 5n$$

$$\frac{6(4n + 3)}{-5} = 5n$$

$$6(4n + 3) = -25n$$

$$24n + 18 = -25n$$

$$24n + 25n = -18$$

$$49n = -18$$

$$\therefore n = -\frac{18}{49}$$

15. **(i)** Let the smallest odd number be n.

The next odd number will be $n + 2$.

The greatest odd number will be $(n + 2) + 2 = n + 4$.

$$\therefore S = n + (n + 2) + (n + 4)$$

$$= n + n + n + 2 + 4$$

$$= 3n + 6$$

(ii) When the greatest odd number is -101,

$$n + 4 = -101$$

$$\therefore n = -101 - 4$$

$$= -105$$

$$\therefore S = 3(-105) + 6$$

$$= -309$$

16. **(i)** $T = c \times d + e \times \frac{f}{100}$

$$= cd + \frac{ef}{100}$$

(ii) $e = \dfrac{-145c}{4-c}$

When $e = 150$,

$$150 = \frac{-145c}{4-c}$$
$$150(4-c) = -145c$$
$$600 - 150c = -145c$$
$$-150c + 145c = -600$$
$$-5c = -600$$
$$\therefore c = \frac{-600}{-5}$$
$$= 120$$

$d = \dfrac{f+5}{50}$

When $d = 3$,

$$3 = \frac{f+5}{50}$$
$$50 \times 3 = f + 5$$
$$150 = f + 5$$
$$f + 5 = 150$$
$$\therefore f = 150 - 5$$
$$= 145$$
$$\therefore T = 120(3) + \frac{150(145)}{100}$$
$$= 577.50$$

17. $y = (x - 32) \times \dfrac{5}{9}$

(i) When $x = 134$,

$$y = (134 - 32) \times \frac{5}{9}$$
$$= 56.7 \text{ (to 3 s.f.)}$$

Required temperature = 56.7 °C

(ii) When $x = 0$,

$$y = (0 - 32) \times \frac{5}{9}$$
$$= -17.8 \text{ (to 3 s.f.)}$$

Since 0 °F = –17.8 °C, it is less common for the temperature to fall below 0 °F because 0 °F is much lower than 0 °C.

(iii) When $y = -62.1$,

$$-62.1 = (x - 32) \times \frac{5}{9}$$
$$(x - 32) \times \frac{5}{9} = -62.1$$
$$x - 32 = -62.1 \times \frac{9}{5}$$
$$x - 32 = -111.78$$
$$\therefore x = -111.78 + 32$$
$$= -79.78$$
$$= -79.8 \text{ (to 3 s.f.)}$$

Required temperature = –79.8 °F

Exercise 6C

1. Let the mass of the empty lorry be x kg.
Then the mass of the bricks is $3x$ kg.

$$x + 3x = 11\,600$$
$$4x = 11\,600$$
$$x = \frac{11\,600}{4}$$
$$= 2900$$

$\therefore$ The mass of the bricks is 3(2900) = 8700 kg.

2. Let the smallest odd number be n.
The next odd number will be $n + 2$.
Then the next odd number will be $(n + 2) + 2 = n + 4$.
The greatest odd number will be $(n + 4) + 2 = n + 6$.

$$n + (n + 2) + (n + 4) + (n + 6) = 56$$
$$n + n + n + n + 2 + 4 + 6 = 56$$
$$4n + 12 = 56$$
$$4n = 56 - 12$$
$$4n = 44$$
$$n = \frac{44}{4}$$
$$= 11$$

$\therefore$ The greatest of the 4 numbers is 11 + 6 = 17.

3. Let Seema's age be x years old.
Then Amirah's age is $(x + 4)$ years,
Sarah's age is $(x - 2)$ years.

$$x + (x + 4) + (x - 2) = 47$$
$$x + x + x + 4 - 2 = 47$$
$$3x + 2 = 47$$
$$3x = 47 - 2$$
$$3x = 45$$
$$x = \frac{45}{3}$$
$$= 15$$

$\therefore$ Seema is 15 years old, Amirah is 15 + 4 = 19 years old and Sarah is 15 – 2 = 13 years old.

4. Let the greater number be x.
Then the smaller number is $\dfrac{2}{3}x$.

$$x + \frac{2}{3}x = 45$$
$$\frac{5}{3}x = 45$$
$$x = \frac{3}{5} \times 45$$
$$= 27$$

$\therefore$ The smaller number is $\dfrac{2}{3}(27) = 18$.

5. Let the number be x.

$$3x = x + 28$$
$$3x - x = 28$$
$$2x = 28$$
$$\therefore x = \frac{28}{2}$$
$$= 14$$

The number is 14.

6. Let x be the number of people going on holiday.

$$16\,200x = 243\,000$$
$$x = \frac{243\,000}{16\,200}$$
$$x = 15$$

15 people are going on holiday.

7. Let x be the number of boys who play badminton.

$$3x + x = 60$$
$$4x = 60$$
$$x = \frac{60}{4}$$
$$x = 15$$

15 boys play badminton.

8. Let the number be x.

$$\frac{1}{2}x + 49 = \frac{9}{4}x$$
$$\frac{1}{2}x - \frac{9}{4}x = -49$$
$$-\frac{7}{4}x = -49$$
$$\therefore x = -\frac{4}{7} \times (-49)$$
$$= 28$$

The number is 28.

9. Let x be the number.

$$68 - 4x = 72$$
$$-4x = 72 - 68$$
$$-4x = 4$$
$$x = \frac{-4}{4}$$
$$x = -1$$

10. Let the son's age be x years.

Then the man's age is $6x$ years.

$$6x + 20 = 2(x + 20)$$
$$6x + 20 = 2x + 40$$
$$6x - 2x = 40 - 20$$
$$4x = 20$$
$$x = \frac{20}{4}$$
$$= 5$$

$\therefore$ The man was $6(5) - 5 = 25$ years old when his son was born.

11. Let the cost of a candy. Then the cost of a jelly is PKR $(x + 2)$.

$$6(x + 2) + 5x = 130.8$$
$$6x + 12 + 5x = 130.8$$
$$6x + 5x = 130.8 - 12$$
$$11x = 118.8$$
$$x = \frac{118.8}{11}$$
$$= 10.8$$

$\therefore$ The cost of a jelly is

PKR $(10.8 + 2)$ = PKR 12.80.

12. Let the number of PKR 20 coins Salman has be x.

Then the number of PKR 10 coins he has is $x + 12$.

$$10(x + 12) + 20x = 540$$
$$10x + 120 + 20x = 540$$
$$10x + 20x = 540 - 120$$
$$30x = 420$$
$$x = \frac{420}{30}$$
$$= 14$$

$\therefore$ Salman has $14 + (14 + 12) = 40$ coins.

13. Let Kiran's average speed for the first part of her journey be x km/h. Then her average speed for the second part of her journey is $(x - 15)$ km/h.

Time taken for first part of journey $= \frac{350}{x}$ hours.

Time taken for second part of journey $= \frac{470 - 350}{x - 15}$

$= \frac{120}{x - 15}$ hours.

$$\frac{350}{x} = \frac{120}{x - 15}$$
$$350(x - 15) = 120x$$
$$350x - 5250 = 120x$$
$$350x - 120x = 5250$$
$$230x = 5250$$
$$x = \frac{5250}{230}$$
$$= 22\frac{19}{23}$$

$\therefore$ Kiran's average speed for the second part of her journey is

$22\frac{19}{23} - 15 = 7\frac{19}{23}$ km/h.

14. Let the denominator of the fraction be x.

Then the numerator of the fraction is $x - 5$.

$\therefore$ The fraction is $\dfrac{x-5}{x}$.

$$\frac{x-5+1}{x+1} = \frac{2}{3}$$
$$\frac{x-4}{x+1} = \frac{2}{3}$$
$$3(x-4) = 2(x+1)$$
$$3x - 12 = 2x + 2$$
$$3x - 2x = 2 + 12$$
$$x = 14$$

$\therefore$ The fraction is $\dfrac{14-5}{14} = \dfrac{9}{14}$.

15. Let the number in the tens place be x.

Then the number in the ones place is $2.5x$.

$\therefore$ The number is $10x + 2.5x = 12.5x$.

$\therefore$ The number obtained when the digits are reversed is $10(2.5x) + x = 25x + x = 26x$.

$$26x - 12.5x = 27$$
$$13.5x = 27$$
$$x = \frac{27}{13.5}$$
$$= 2$$

The number is $12.5(2) = 25$.

Review Exercise 6

1. **(a)**
$$x - 1 = \frac{1}{2}x$$
$$x - \frac{1}{2}x = 1$$
$$\frac{1}{2}x = 1$$
$$\therefore x = 2$$

(b)
$$2(x-1) + 3(x+1) = 4(x+4)$$
$$2x - 2 + 3x + 3 = 4x + 16$$
$$5x + 1 = 4x + 16$$
$$5x - 4x = 16 - 1$$
$$\therefore x = 15$$

(c)
$$2y - [7 - (5y - 4)] = 6$$
$$2y - (7 - 5y + 4) = 6$$
$$2y - (11 - 5y) = 6$$
$$2y - 11 + 5y = 6$$
$$7y - 11 = 6$$
$$7y = 6 + 11$$
$$7y = 17$$
$$\therefore y = \frac{17}{7}$$
$$= 2\frac{3}{7}$$

(d)
$$\frac{3}{4}x - 5 = 0.5x$$
$$\frac{3}{4}x - 0.5x = 5$$
$$\frac{1}{4}x = 5$$
$$\therefore x = 4 \times 5$$
$$= 20$$

(e)
$$\frac{2y+7}{4} = 12$$
$$2y + 7 = 4 \times 12$$
$$2y + 7 = 48$$
$$2y = 48 - 7$$
$$2y = 41$$
$$\therefore y = \frac{41}{2}$$
$$= 20\frac{1}{2}$$

(f)
$$\frac{4y-1}{5y+1} = \frac{5}{7}$$
$$7(4y-1) = 5(5y+1)$$
$$28y - 7 = 25y + 5$$
$$28y - 25y = 5 + 7$$
$$3y = 12$$
$$\therefore y = \frac{12}{3}$$
$$= 4$$

(g)
$$\frac{a+1}{4} + \frac{a-1}{3} = 4$$
$$\frac{3(a+1) + 4(a-1)}{12} = 4$$
$$\frac{3a + 3 + 4a - 4}{12} = 4$$
$$\frac{7a-1}{12} = 4$$
$$7a - 1 = 12 \times 4$$
$$7a - 1 = 48$$
$$7a = 48 + 1$$
$$7a = 49$$
$$\therefore a = \frac{49}{7}$$
$$= 7$$

(h) $\frac{b-4}{3} - \frac{2b+1}{6} = \frac{5b-1}{2}$

$\frac{2(b-4)-(2b+1)}{6} = \frac{5b-1}{2}$

$\frac{2b-8-2b-1}{6} = \frac{5b-1}{2}$

$\frac{-9}{6} = \frac{5b-1}{2}$

$\frac{-3}{2} = \frac{5b-1}{2}$

$-3 = 5b-1$

$5b-1 = -3$

$5b = -2$

$\therefore b = -\frac{2}{5}$

(i) $\frac{2c}{9} - \frac{c-1}{6} = \frac{c+3}{12}$

$\frac{2(2c)-3(c-1)}{18} = \frac{c+3}{12}$

$\frac{4c-3c+3}{18} = \frac{c+3}{12}$

$\frac{c+3}{18} = \frac{c+3}{12}$

$12(c+3) = 18(c+3)$

$2(c+3) = 3(c+3)$

$2c+6 = 3c+9$

$2c-3c = 9-6$

$-c = 3$

$\therefore c = -3$

(j) $\frac{2(3-4d)}{3} - \frac{3(d+7)}{2} = 5d + \frac{1}{6}$

$\frac{4(3-4d)-9(d+7)}{6} = \frac{6(5d)+1}{6}$

$\frac{12-16d-9d-63}{6} = \frac{30d+1}{6}$

$\frac{-25d-51}{6} = \frac{30d+1}{6}$

$-25d-51 = 30d+1$

$-25d-30d = 1+51$

$-55d = 52$

$\therefore d = -\frac{52}{55}$

2. $V = \frac{4}{3}\pi r^3$

(i) When $r = 7$,

$V = \frac{4}{3}\ \frac{22}{7}\ (7)^3$

$= 1437\frac{1}{3}$

(ii) When $V = 113\frac{1}{7}$,

$113\frac{1}{7} = \frac{4}{3}\ \frac{22}{7}\ r^3$

$113\frac{1}{7} = \frac{88}{21}r^3$

$\frac{88}{21}r^3 = 113\frac{1}{7}$

$r^3 = \frac{21}{88} \times 113\frac{1}{7}$

$r^3 = 27$

$\therefore r = \sqrt[3]{27}$

$= 3$

3. $n - 2y = \frac{3y-n}{m}$

When $y = 5$, $m = -3$,

$n - 2(5) = \frac{3(5)-n}{-3}$

$n - 10 = \frac{15-n}{-3}$

$-3(n-10) = 15-n$

$-3n+30 = 15-n$

$-3n+n = 15-30$

$-2n = -15$

$\therefore n = \frac{-15}{-2}$

$= 7\frac{1}{2}$

4. Let the smaller odd number be n.

Then the greater number is $n + 2$.

$n + 2 + 5n = 92$

$6n + 2 = 92$

$6n = 92 - 2$

$6n = 90$

$n = \frac{90}{6}$

$= 15$

$\therefore$ The two consecutive odd numbers are 15 and 17.

5. Let the mass of Object B be x kg.

Then the mass of Object A is $(x + 5)$ kg,

the mass of Object C is $2(x + 5)$ kg.

$(x+5) + x + 2(x+5) = 255$

$x + 5 + x + 2x + 10 = 255$

$4x + 15 = 255$

$4x = 255 - 15$

$4x = 240$

$x = \frac{240}{4}$

$= 60$

$\therefore$ The mass of Object C is $2(60 + 5) = 130$ kg.

6. Let Farhan's present age be x years.

Then Farhan's cousin's present age is $(38 - x)$ years.

$$\begin{aligned} x - 7 &= 3(38 - x - 7) \\ x - 7 &= 3(31 - x) \\ x - 7 &= 93 - 3x \\ x + 3x &= 93 + 7 \\ 4x &= 100 \\ \therefore x &= \frac{100}{4} \\ &= 25 \end{aligned}$$

Farhan is 25 years old now.

7. Let x be Saad's present age.

Nadia's age is given by $2x$.

$$\begin{aligned} 2x + x &= 21 \\ 3x &= 21 \\ x &= \frac{21}{3} \\ x &= 7 \end{aligned}$$

Nadia's age is $2 \times 7 = 14$ years

After 22 years, Nadia's age will be $14 + 22 = 36$ years.

8. Let the number of sweets that the man has to give to his son be x.

$$\begin{aligned} 55 + x &= 4(25 - x) \\ 55 + x &= 100 - 4x \\ x + 4x &= 100 - 55 \\ 5x &= 45 \\ \therefore x &= \frac{45}{5} \\ &= 9 \end{aligned}$$

The man has to give 9 sweets to his son.

9. Let the original price of each apple be x cents.

$$\begin{aligned} 24x &= (24 + 6)(x - 5) \\ 24x &= 30(x - 5) \\ 24x &= 30x - 150 \\ 24x - 30x &= -150 \\ -6x &= -150 \\ \therefore x &= \frac{-150}{-6} \\ &= 25 \end{aligned}$$

The original price of each apple is 25 cents.

10. Let the distance between Town A and Town B be x km.

45 minutes $= \frac{45}{60}$ hour $= \frac{3}{4}$ hour

$$\begin{aligned} \frac{x}{4} + \frac{x}{6} &= \frac{3}{4} \\ \frac{6x + 4x}{24} &= \frac{3}{4} \\ \frac{10x}{24} &= \frac{3}{4} \\ \frac{5x}{12} &= \frac{3}{4} \\ 5x &= 12 \times \frac{3}{4} \\ 5x &= 9 \\ x &= \frac{9}{5} \\ &= 1\frac{4}{5} \end{aligned}$$

$\therefore$ The main travels a total distance of $2 \times 1\frac{4}{5} = 3\frac{3}{5}$ km.

Challenge Yourself

1.
$$\begin{aligned} \sqrt{x} + 2 &= 0 \\ \sqrt{x} &= -2 \end{aligned}$$

There is no solution since $\sqrt{x}$ cannot be a negative number.

2.
$A + B = 8$ — (1)

$B + C = 11$ — (2)

$B + D = 13$ — (3)

$C + D = 14$ — (4)

(2) – (3): $B + C - B - D = 11 - 13$

$C - D = -2$ — (5)

(4) + (5): $C + D + C - D = 14 + (-2)$

$$\begin{aligned} 2C &= 12 \\ \therefore C &= \frac{12}{2} \\ &= 6 \end{aligned}$$

Substitute $C = 6$ into (4): $6 + D = 14$

$$\begin{aligned} \therefore D &= 14 - 6 \\ &= 8 \end{aligned}$$

Substitute $C = 6$ into (2): $B + 6 = 11$

$$\begin{aligned} \therefore B &= 11 - 6 \\ &= 5 \end{aligned}$$

Substitute $B = 5$ into (1): $A + 5 = 8$

$$\begin{aligned} \therefore A &= 8 - 5 \\ &= 3 \end{aligned}$$

Chapter 7 Rate and Ratio

TEACHING NOTES

Suggested Approach

Students will learn how to solve problems involving ratios and rate. Teachers can bring in real-life examples for ratio and rate to arouse students' interest in this topic. Students will also learn how to solve problems involving ratio and rate through worked examples that involve situations in real-world contexts.

Section 7.1: Ratio

Teachers can build upon what students have learnt about ratio in primary school and introduce equivalent ratios through a recap of equivalent fractions. Teachers should emphasise that ratio does not indicate the actual size of quantities involved. Practical examples can be given to the students to let them recognise what equivalent ratios are (e.g. using 2 different kinds of fruits).

Teachers should highlight some common errors in ratio (i.e. the ratio of a part of a whole with the ratio of two parts, incorrect order of numbers expressed when writing ratio and incorrect numerator expressed when writing ratio as a fraction).

To make learning interesting, students can explore more about the Golden Ratio (see chapter opener and Investigation: Golden Ratio). Teachers can also get the students to find out what other man-made structures or natural occurrences have in common with the Golden Ratio (see Performance Task at page 125 of the textbook).

Section 7.2: Rate

Teachers should explain that rate is a relationship between two quantities with different units of measure (which is different from ratio). Teachers can give real life examples (e.g. rate of flow, consumption) for students to understand the concept of rate. Teachers can also get students to interpret using tables which show different kinds of rates (e.g. interest rate, postage rate, parking rate etc.).

Students can get more practice by learning to calculate rates they are familiar with. Teachers should impress upon them to distinguish between constant and average rates.

Challenge Yourself

Teachers can guide the students by getting them to use appropriate algebraic variables to represent the rates involved in the question. Students have to read the question carefully and form the linear equations which then can be solved to get the answers.

WORKED SOLUTIONS

Class Discussion (Making Sense of the Relationship between Ratios and Fractions)

There are 40 green balls and 60 red balls in a bag.

Let A and B represent the number of green balls and red balls respectively.

1. Find the ratio of A to B.

$A : B = 40 : 60$

$= \underline{2} : \underline{3}$

We can conclude that:

The ratio of A to B is $\underline{2} : \underline{3}$.

The following statement is equivalent to the above statement.

A is $\underline{\frac{2}{3}}$ (fraction) of B, i.e. $\frac{A}{B} = \underline{\frac{2}{3}}$ (fraction).

2. Find the ratio of B to A.

$B : A = 60 : 40$

$= \underline{3} : \underline{2}$

We can conclude that:

The ratio of B to A is $\underline{3} : \underline{2}$.

The following statement is equivalent to the above statement.

B is $\underline{\frac{3}{2}}$ (fraction) of A, i.e. $\frac{B}{A} = \underline{\frac{3}{2}}$ (fraction).

3.

A	20	20	
B	20	20	20

4. Example:

There are 30 girls and 10 boys in a class.

Let G and B represent the number of girls and boys respectively.

$G : B = 30 : 10$

$= 3 : 1$

We can conclude that:

The ratio of G to B is $3 : 1$.

The following statement is equivalent to the above statement.

G is $\frac{3}{1}$ (fraction) of B, i.e. $\frac{G}{B} = \frac{3}{1}$ (fraction).

OR

$B : G = 10 : 30$

$= 1 : 3$

We can conclude that:

The ratio of B to G is $1 : 3$.

The following statement is equivalent to the above statement.

B is $\frac{1}{3}$ (fraction) of G, i.e. $\frac{B}{G} = \frac{1}{3}$ (fraction).

G	10	10	10
B	10		

Journal Writing (Page 124)

1. Aspect ratio is used to describe the relationship between the width and height of an image. It does not represent the actual length and height, but instead represents the proportion of its width and height. This is usually represented by two numbers separated by a colon, for example, 4 : 3 and 16 : 9.

The standard size of televisions has an aspect ratio of 4 : 3 which means the image is 4 units wide for every 3 units of height. Meanwhile, the latest size of televisions for the aspect ratio is 16 : 9 which is 16 units of width for every 9 units of height.

The following are some examples of aspect ratio used in our daily lives:

- 16 : 10 is used mainly in widescreen computer monitors.
- 16 : 9 is the aspect ratio used in cinema halls as well as High Definition TV.
- 14 : 9 is a compromise aspect ratio used to create an image that is viewable to both 4 : 3 and 16 : 9 televisions.
- 5 : 4 is a computer monitor resolution and also in mobile phones.
- 4 : 3 is used in the older TVs (mainly non-widescreen) and computer monitors.
- 1 : 1 is an uncommon aspect ratio that is used mainly in photography.

2. Example 1:

Scale drawings of maps and buildings are often represented by ratios. This is because it is impossible for a map to be exactly of the same size as the area it represents. Therefore, the measurements are scaled down in a fixed proportion so that the map can be used easily. Similarly, a scale drawing of a building will have the same shape as the actual building except that is scaled down.

Example 2:

In Chemistry and Biology, ratios are used for simple dilution of chemicals. A fixed unit volume of a chemical is added to an appropriate volume of solvent in order to dilute the chemical. For example, a 1 : 5 dilution (verbalize as “1 to 5” dilution) entails combining 1 unit volume of solute (the material to be diluted) + 4 unit volumes (approximately) of the solvent to give 5 units of the total volume.

Investigation (Golden Ratio)

1. $AB = 1.7$ cm

$BC = 1.05$ cm

$\frac{AC}{AB} = \frac{2.75}{1.7} = 1.62$ (to 2 s.f.)

$\frac{AB}{BC} = \frac{1.7}{1.05} = 1.62$ (to 2 s.f.)

2. $XY = 2.75$ cm

$YZ = 1.7$ cm

$\frac{XY}{YZ} = \frac{2.75}{1.7} = 1.62$ (to 2 s.f.)

3. $\frac{1 + \sqrt{5}}{2} = 1.62$ (to 2 s.f.)

4. All the values in the previous questions are all equal.

5. –

6. **(a)** $\varphi^2 = \dfrac{3+\sqrt{5}}{2}$

$\varphi + 1 = \dfrac{3+\sqrt{5}}{2}$

Both answers are the same.

(b) $\dfrac{1}{\varphi} = \dfrac{-1+\sqrt{5}}{2}$

$\dfrac{1}{\varphi} = \varphi - \_\_\_\_$

$\varphi - \dfrac{1}{\varphi} = \dfrac{1+\sqrt{5}}{2} - \dfrac{-1+\sqrt{5}}{2}$

$= 1$

It is equal to 1.

Performance Task (Page 125)

Teachers may wish to give some examples of

- man-made structures such as the
  - **a)** Acropolis of Athens (468–430 BC), including the Parthenon;
  - **b)** Great Mosque of Kairouan (built by Uqba ibn Nafi c. 670 A.D);
  - **c)** Cathedral of Chartres (begun in the 12th century), Notre-Dame of Laon (1157–1205), and Notre Dame de Paris (1160);
  - **d)** Mexico City Metropolitan Cathedral (1667–1813).
- natural occurrences
  - **a)** spiral growth of sea shells;
  - **b)** spiral of a pinecone;
  - **c)** petals of sunflower;
  - **d)** horns of antelopes, goats and rams;
  - **e)** tusks of elephants;
  - **f)** body dimensions of penguins.

Investigation (Average Pulse Rate)

	First reading	Second reading	Third reading
Pulse rate (per minute)			

Thinking Time (Page 129)

1. The parking charges per minute are PKR 0.40 is a constant rate as the rate of charges per minute is the same throughout.
The rate of petrol consumption is 13.5 km per litre is an average rate as the rate of consumption is not the same per minute.

2. The following are 3 examples of average rate that can be found in daily life:

- Average speed
- Downloading rate of a file
- Average daily population growth

The following are 3 example of constant rate that can be found in daily life:

- Simple interest rate
- Income Tax rate
- Currency exchange rate

Practise Now 1

(i) Ratio of the number of lemons to the number of pears
$= 33 : 20$

(ii) Ratio of the number of pears to the number of fruits in the basket
$= 20 : (33 + 20)$
$= 20 : 53$

Practise Now 2

(a) $240 \text{ g} : 1.8 \text{ kg} = 240 \text{ g} : 1800 \text{ g}$
$= 2 : 15$

Alternatively,

$\dfrac{2.40 \text{ g}}{1.8 \text{ kg}} = \dfrac{240 \text{ g}}{1800 \text{ g}}$

$= \dfrac{2}{15}$

$\therefore 240 \text{ g} : 1.8 \text{ kg} = 2 : 15$

(b) $\dfrac{3}{5} : \dfrac{8}{9} = \dfrac{3}{5} \times 45 : \dfrac{8}{9} \times 45$
$= 27 : 40$

(c) $0.36 : 1.2 = 0.36 \times 100 : 1.2 \times 100$
$= 36 : 120$
$= 3 : 10$

Practise Now 3

$3a : 7 = 8 : 5$

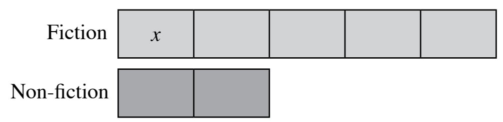
$\dfrac{3a}{7} = \dfrac{8}{5}$

$15a = 56$

$a = 3\dfrac{11}{15}$

Practise Now 4

1. Let the number of fiction books = $5x$.
Then the number of non-fiction book = $2x$.



From the model, we form the equation:

$5x + 2x = 1421$

$7x = 1421$

$x = 203$

There are $3 \times 203 = 609$ more fiction than non-fiction books in the library.

2. Let the amount of money Kate had initially be PKR $3x$.
Then the amount of money Nora had initially is PKR $5x$.

	Kate	Nora
Before	PKR $3x$	PKR $5x$
After	PKR $(3x + 150)$	PKR $(5x - 150)$

$$\therefore \frac{3x + 150}{5x - 150} = \frac{7}{9}$$
$$9(3x + 150) = 7(5x - 150)$$
$$27x + 1350 = 35x - 1050$$
$$27x - 35x = -1050 - 1350$$
$$-8x = -2400$$
$$x = 300$$
$\therefore$ Amount of money Kate had initially
= PKR [3(300)]
= PKR 900

Practise Now 5

Number of words per minute that Amirah can type
$= \frac{720}{16}$
= 45
Number of words per minute that Faiza can type
$= \frac{828}{18}$
= 46
Number of words per minute that Sara can type
$= \frac{798}{19}$
= 42
Thus, Faiza is the fastest typist.

Practise Now 6

1. (a) Amount each child have to pay
$= \frac{\text{PKR } 270 \times 32.5}{36}$
= PKR 244
(b) (i) Distance travelled on 1 litre of petrol
$= \frac{265}{25}$
$= \frac{265}{25}$
= 10.6 km
Distance travelled on 58 litres of petrol
$= 10.6 \times 58$
= 614.8 km
(ii) Amount of petrol required to travel a distance of 1007 km
$= \frac{1007}{10.6}$
= 95 litres
Amount that the car owner has to pay
$= 95 \times$ PKR 195
= PKR 18 525

2. In 1 minute, 5 people can finish
$= 20 \div 3\frac{20}{60}$
= 6 buns
In 5 minutes, 5 people can finish
$= 6 \times 5$
= 30 buns
In 5 minutes, 10 people can finish
$= 30 \times 2$
= 60 buns

Exercise 7A

1. (a) 1.5 kg : 350 g = 1500 g : 350 g
= 30 : 7
Alternatively,
$$\frac{1.5 \text{ kg}}{350 \text{ g}} = \frac{1500 \text{ g}}{350 \text{ g}}$$
$$= \frac{30}{7}$$
$\therefore$ 1.5 kg : 350 g = 30 : 7
(b) $\frac{15}{24} : \frac{9}{7} = \frac{15}{24} \times 168 : \frac{9}{7} \times 168$
= 105 : 216
= 35 : 72
(c) $0.45 : 0.85 = 0.45 \times 100 : 0.85 \times 100$
= 45 : 85
= 9 : 17
(d) 580 ml : 1.12 l : 104 ml = 580 : 1120 : 104
= 145 : 280 : 26
(e) $\frac{2}{3} : \frac{3}{2} : \frac{5}{8} = \frac{2}{3} \times 24 : \frac{3}{2} \times 24 : \frac{5}{8} \times 24$
= 16 : 36 : 15
(f) $0.33 : 0.63 : 1.8 = 0.33 \times 100 : 0.63 \times 100 : 1.8 \times 100$
= 33 : 63 : 180
= 11 : 21 : 60

2. (a) $a : 400 = 6 : 25$
$$\frac{a}{400} = \frac{6}{25}$$
$$25a = 2400$$
$$a = 96$$
(b) $5b : 8 = 2 : 5$
$$\frac{5b}{8} = \frac{2}{5}$$
$$25b = 16$$
$$b = \frac{16}{25}$$

3. $$\frac{2x}{5} = \frac{3y}{8}$$
$$16x = 15y$$
$$\frac{x}{y} = \frac{15}{16}$$
$x : y = 15 : 16$

4. **(i)** Ratio of the number of boys to the number of girls = 14 : 25

(ii) Ratio of the number of girls to the total number of players in the team = 25 : 39

5. **(i)** Ratio of the number of athletes to the number of volunteers

= 3600 : 20 000

= 9 : 50

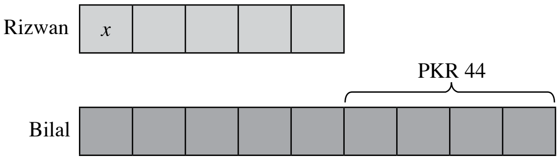
(ii) Ratio of the number of media representatives to the number of athletes to the number of spectators

= 1200 : 3600 : 370 000

= 3 : 9 : 925

6. Let the amount of money that Rizwan gets = $5x$.

Then the amount of money that Bilal gets = $9x$.



From the model, we form the equation:

$9x - 5x = 44$

$4x = 44$

$x = 11$

Total amount of money that is shared between the two boys

= (5 + 9) × 11

= PKR 154

7. **(i)** Number of toys Anusha makes $= \frac{560}{12 + 16} \times 16$

$= \frac{560}{28} \times 16$

$= 320$

(ii) Number of toys Amirah makes $= \frac{560}{12 + 16} \times 12$

$= \frac{560}{28} \times 12$

$= 240$

Amount of money Amirah earns = 240 × PKR 165

= PKR 39 600

8. **(a)** $4\frac{1}{5}$ kg : 630 g = 4200 g : 630 g

= 20 : 3

(b) $0.75 : 3\frac{5}{16} = \frac{75}{100} : 3\frac{5}{16}$

$= \frac{3}{4} : \frac{53}{16}$

$= \frac{3}{4} \times 16 : \frac{53}{16} \times 16$

$= 12 : 53$

(c) 0.6 kg : $\frac{3}{4}$ kg : 400 g = 600 g : 750 g : 400 g

= 12 : 15 : 8

(d) $\frac{1}{3} : 2.5 : 3\frac{3}{4} = \frac{1}{3} \times 12 : 2.5 \times 12 : \frac{15}{4} \times 12$

$= 4 : 30 : 45$

(e) $1.2 : 3\frac{3}{10} : 5.5 = 1.2 \times 10 : \frac{33}{10} \times 10 : 5.5 \times 10$

$= 12 : 33 : 55$

9. **(a)** $2\frac{1}{4} : 6 = m : 1\frac{1}{5}$

$\frac{9}{4} : 6 = m : \frac{6}{5}$

$\frac{9}{4} \times 20 : 6 \times 20 = m \times 20 : \frac{6}{5} \times 20$

$45 : 120 = 20m : 24$

$9 : 24 = 20m : 24$

$9 = 20m$

$20m = 9$

$m = \frac{9}{20}$

(b) $x : 3 : \frac{9}{2} = \frac{15}{4} : 4\frac{1}{2} : y$

$x \times 4 : 3 \times 4 : \frac{9}{2} \times 4 = \frac{15}{4} \times 4 : \frac{9}{2} \times 4 : y \times 4$

$4x : 12 : 18 = 15 : 18 : 4y$

$\frac{4x}{12} = \frac{15}{18}$

$\frac{x}{3} = \frac{5}{6}$

$6x = 15$

$x = \frac{15}{6}$

$x = \frac{5}{2}$

$x = 2\frac{1}{2}$

$\frac{12}{18} = \frac{18}{4y}$

$\frac{2}{3} = \frac{9}{2y}$

$4y = 27$

$y = \frac{27}{4}$

$y = 6\frac{3}{4}$

10. $p : q = \frac{3}{4} : 2$

$\downarrow \times 4$

$= 3 : 8$

$\downarrow$

$= 6 : 16$

$p : r = \frac{1}{3} : \frac{1}{2}$

$\downarrow \times 6$

$= 2 : 3$

$\downarrow$

$= 6 : 9$

(i) $p : q : r = 6 : 16 : 9$

(ii) $q : r = 16 : 9$

11. **(i)** Let the initial number of teachers in the school be x.

Then the number of students in the school is $15x$.

$15x = 1200$

$x = 80$

The initial number of teachers in the school is 80.

(ii) Let the number of teachers who join the school be y.

$\frac{80 + y}{1200} = \frac{3}{40}$

$40(80 + y) = 3(1200)$

$3200 + 40y = 3600$

$40y = 3600 - 3200$

$40y = 400$

$y = 10$

The number of teachers who join the school is 10.

12. Ratio of Ahsan, Farhan and Michael's property investment

= PKR 427 000 : PKR 671 000 : PKR 305 000

= 427 : 671 : 305

= 7 : 11 : 5

Total amount of profit earned

= PKR 1 897 500 – (PKR 427 000 + PKR 671 00 + PKR 305 000)

= PKR 494 500

Amount of profit Ahsan received

$= \frac{7}{7+11+5} \times$ PKR 494 500

= PKR 150 000

Amount of profit Farhan received

$= \frac{11}{7+11+5} \times$ PKR 494 500

= PKR 236 500

Amount of profit Michael received

$= \frac{5}{7+11+5} \times$ PKR 494 500

= PKR 107 500

13. Let the number that must be added be x.

$$\frac{3+x}{8+x} = \frac{2}{3}$$
$$3(3+x) = 2(8+x)$$
$$9+3x = 16+2x$$
$$3x-2x = 16-9$$
$$x = 7$$

The number is 7.

14. Let the amount of money Ahsan had initially be PKR $5x$.

Then the amount of money Salman and Raj had initially is PKR $6x$ and PKR $9x$ respectively.

	Ahsan	Salman	Raj
Before	PKR $5x$	PKR $6x$	PKR $9x$
After	PKR $(5x – 50)$	PKR $6x$	PKR $9x$

$$\therefore \frac{5x-50}{6x} = \frac{3}{4}$$
$$4(5x-50) = 3(6x)$$
$$20x-200 = 18x$$
$$20x-18x = 200$$
$$2x = 200$$
$$x = 100$$

$\therefore$ Amount of money Ahsan has after giving PKR 50 to his mother

= PKR [5(100) – 50]

= PKR 450

15. $\frac{x}{y} = \frac{3}{4}$ $\qquad$ $\frac{y}{z} = \frac{5}{8}$

$4x = 3y$ $\qquad$ $8y = 5z$

$x = \frac{3}{4}y$ $\qquad$ $z = \frac{8}{5}y$

$$\frac{2y}{3x-y+2z} = \frac{2y}{3\ \frac{3}{4}y\ -y+2\ \frac{8}{5}y}$$
$$= \frac{2y}{\frac{9}{4}y-y+\frac{16}{5}y}$$
$$= \frac{2y}{\frac{45}{20}y-\frac{20}{20}y+\frac{64}{20}y}$$
$$= \frac{2y}{\frac{89}{20}y}$$
$$= \frac{40}{89}$$

Exercise 7B

1. **(a)** Number of words that she can type per minute

$= \frac{1800}{60}$

(1 hour = 60 minutes)

= 30

(b) Cost of one unit of electricity

= PKR $\frac{120.99}{654}$

= PKR 0.19

(c) His monthly rental rate

= PKR $\frac{48\ 000}{3}$

= PKR 16 000

(d) Its mass per metre

$= \frac{15}{3.25}$

$= 4\frac{8}{13}$ kg/m

2. Time taken for Ahsan to blow 1 balloon

$= \frac{20}{15}$

$= 1.\dot{3}$ minutes

Time taken for Salman to blow 1 balloon

$= \frac{25}{18}$

$= 1.3\dot{8}$ minutes

Time taken for Bilal to blow 1 balloon

$= \frac{21}{16}$

= 1.3125 minutes

Thus, Bilal can blow balloons at the fastest rate.

3. 3 hours = 180 minutes

Number of ornaments made in 3 hours

$= \frac{180}{15} \times 4$

$= 48$

Amount earned by the worker

= 48 × PKR 1.15

= PKR 55.20

4. **(i)** Amount he is charged for each minute of outgoing calls

= PKR $\frac{39}{650}$

= PKR 0.06

(ii) Amount he has to pay

= PKR 0.06 × 460

= PKR 27.60

5. **(i)** Distance travelled on 1 litre of petrol

$= \frac{259.6}{22}$

= 11.8 km

Distance travelled on 63 litres of petrol

$= 11.8 \times 63$

= 743.4 km

(ii) Amount of petrol required to travel a distance of 2013.2 km

$= \frac{2013.2}{11.8}$

$= 170\frac{36}{59}$ litres

Amount that the car owner has to pay

$= 170\frac{36}{59}$ × PKR 199

= PKR 33951.4

6. **(i)** Amount of fertiliser needed for a plot of land that has an area of 1 m^2

$= \frac{200}{8}$

= 25 g

Amount of fertiliser needed for a plot of land that has an area of 14 m^2

$= 25 \times 14$

= 350 g

(ii) Area of land that can be fertilised by 450 g of fertiliser

$= \frac{450}{25}$

= 18 m^2

7. **(i)** Temperature of the metal after 9 minutes

= 428 °C – [(23 °C × 3) + (15 °C × 6)]

= 269 °C

(ii) Temperature of the metal after 18 minutes

= 428 °C – [(23 °C × 3) + (15 °C × 15)]

= 134 °C

Amount of temperature needed for the metal to fall so that it will reach a temperature of 25 °C

= 134 °C – 25 °C

= 109 °C

Time needed for the metal to reach a temperature of 25 °C

$= \frac{109}{8}$

$= 13\frac{5}{8}$ minutes

8. 4 weeks ⇒ fifteen 2-litre bottles of cooking oil

1 week ⇒ $\frac{15 \times 2}{4}$ = 7.5 litres of cooking oil

10 weeks ⇒ 10 × 7.5 = 75 litres of cooking oil

Number of 5-litre tins of cooking oil needed for a 10-week period

$= \frac{75}{5}$

$= 15$

9. **(i)** Total amount to be paid to the man

= 224 × PKR 7.50

= PKR 1680

(ii) Number of normal working hours from 9 a.m. to 6 p.m. excluding lunch time

= 8 hours

Let the number of overtime hours needed to complete the project in 4 days by each worker be x.

$4[4(8 + x)] = 224$

$16(8 + x) = 224$

$128 + 16x = 224$

$16x = 224 - 128$

$16x = 96$

$x = 6$

Overtime hourly rate

= 1.5 × PKR 7.5

= PKR 11.25

Total amount to be paid to the 4 men if the project is to be completed in 4 days

= 4{4[(8 × PKR 7.5) + (6 × PKR 11.25)]}

= PKR 2040

10. 10 chefs can prepare a meal for 536 people in 8 hours and so

1 chef can prepare a meal for 536 people in 8 × 10 = 80 hours.

Hence, 22 chefs can prepare a meal for 536 × 22 = 11 792 people in 80 hours and so 22 chefs can prepare a meal for $\frac{536 \times 22}{80}$ people in $\frac{80}{80}$ = 1 hour.

Thus 22 chefs can prepare a meal for $\frac{536 \times 22}{80} \times 5$ people in 1 × 5 = 5 hours.

$\frac{536 \times 22}{80} \times 5$

$= 737$

Review Exercise 7

1. $a : b = \frac{1}{2} : \frac{1}{3}$ $\quad\quad$ $b : c = 3 : 4$

$\downarrow \times 6$ $\quad\quad$ $\downarrow \times 2$

$= 3 : 2$ $\quad\quad$ $= 6 : 8$

$\downarrow \times 3$

$= 9 : 6$

$\therefore a : c = 9 : 8.$

2. (i) Let the mass of type A coffee beans in the mixture be $3x$ kg.

Then the mass of type B and C coffee beans in the mixture be $5x$ kg and $7x$ kg respectively.

$\therefore 3x + 5x + 7x = 35$

$15x = 45$

$x = 3$

Mass of type A coffee beans in the mixture = $3 \times 3 = 9$ kg

Mass of type B coffee beans in the mixture = $5 \times 3 = 15$ kg

Mass of type C coffee beans in the mixture = $7 \times 3 = 21$ kg

(ii) Cost of the mixture per kg

$= \dfrac{(9 \times \text{PKR } 700) + (15 \times \text{PKR } 1000) + (21 \times \text{PKR } 1300)}{45}$

= PKR 1080

3. (i) Let the number of books in the box be $4x$.

Then the initial number of toys in the box be $5x$.

$\therefore 4x = 36$

$x = 9$

So the initial number of toys in the box is $5 \times 9 = 45$.

(ii) Let the number of toys that are given away be y.

$\therefore \dfrac{36}{45 - y} = \dfrac{12}{11}$

$11(36) = 12(45 - y)$

$396 = 540 - 12y$

$12y = 540 - 396$

$12y = 144$

$y = 12$

The number of toys that are given away is 12.

4. (i) Total cost of placing an advertisement containing 22 words

$= 300.50 + (22 \times 25)$

= PKR 850.5

(ii) Let the number of words he can use be x.

Then $300.5 + 25x \leqslant 1500$

$25x \leqslant 1500 - 300.5$

$25x \leqslant 1199.5$

$x \leqslant 47.398 \simeq 48$

The greatest number of words he can use is 46.

Challenge Yourself

1. Let Hussain's speed be x m/s and Farhan's speed be y m/s.

Then in the first race, when Hussain ran pass the end point 100 m, Farhan is only at 90 m of the race. Hence, at the same time,

$\dfrac{100}{x} = \dfrac{90}{y}$

$100y = 90x$

$x = \dfrac{100y}{90}$

For the second race,

Let the time for the first person to pass the end point be t s.

Time taken for Hussain to finish the 100 m race

$= \dfrac{110}{x}$

$= \dfrac{110}{\frac{100y}{90}}$ (Substitute $x = \dfrac{100y}{90}$)

$= \dfrac{99}{y}$ s

Time taken for Farhan to finish the 100 m race

$= \dfrac{100}{y}$ s

At time t s, distance that Hussain covered

$t = \dfrac{99}{y}$

$ty = 99$ m

At time t s, distance that Farhan covered

$t = \dfrac{100}{y}$

$ty = 100$ m

$\therefore$ Hussain win the race by $100 - 99 = 1$ m.

Chapter 8 Perimeter and Area of Plane Figures

TEACHING NOTES

Suggested Approach

The students will learn how to convert units of area, as well as find the perimeter and area of triangles and quadrilaterals. Students will revise what they have learnt in primary school as well as learn the perimeter and area of parallelograms and trapeziums. Teachers should place more focus on the second half of the chapter and ensure students are able to solve problems involving the perimeter and area of parallelograms and trapeziums.

Section 8.1: Conversion of Units

Teachers may wish to recap with the students the conversion of unit lengths from one unit of measurement to another (i.e. mm, cm, m and km) before moving onto the conversion of units for areas.

Teachers may ask students to remember simple calculations such as 1 cm^2 = 1 cm × 1 cm to help them in their calculations when they solve problems involving the conversion of units.

Section 8.2: Perimeter and Area of Basic Plane Figures

This section is a recap of what students have learnt in primary school. Students are reminded to be clear of the difference in the units used for perimeter and area (e.g. cm and cm^2).

Section 8.3: Perimeter and Area of Parallelograms

Teachers should illustrate the dimensions of a parallelogram to the students so that they are able to identify the base and height of parallelograms. It is important to emphasise to the students that the height of a parallelogram is with reference to the base and it must be perpendicular to the base chosen. Also, the height may lie within, or outside of the parallelogram. Teachers can highlight to the students that identifying the height of a parallelogram is similar to identifying the height of a triangle.

Teachers should guide students in finding the formula for the area of a parallelogram (see Investigation: Formula for Area of a Parallelogram). Both possible methods should be shown to students (The second method involves drawing the diagonal of the parallelogram and finding the area of the two triangles).

Section 8.4: Perimeter and Area of Trapeziums

Teachers should recap with students the properties of a trapezium. Unlike the parallelogram, the base of the trapezium is not required and the height must be with reference to the two parallel sides of the trapezium. Thus, the height lies either inside the trapezium, or it is one of its sides (this occurs in a right trapezium, where two adjacent angles are right angles).

Teachers should guide students in finding the formula for the area of a trapezium (see Investigation: Formula for Area of a Trapezium). Both possible methods should be shown to students (Again, the second method involves drawing the diagonal of the trapezium and finding the area of the two triangles).

Teachers can enhance the students' understanding and appreciation of the areas of parallelograms and trapeziums by showing them the link between the area of a trapezium, a parallelogram and a triangle (see Thinking Time on page 144).

WORKED SOLUTIONS

Class Discussion (International System of Units)

1. The seven basic physical quantities and their base units are shown in the following table:

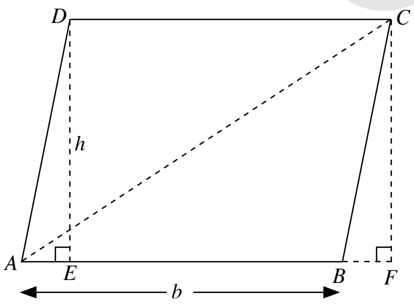
Basic Physical Quantity	Base Unit
Length	metre (m)
Mass	kilogram (kg)
Time	second (s)
Electric current	ampere (A)
Thermodynamic Temperature	kelvin (K)
Amount of substance	mole (mol)
Luminous intensity	candela (cd)

Scientists developed the International System of Units (SI units) so that there is a common system of measures which can be used worldwide.

2. Measurements of Lengths:
1 foot (ft) = 0.3048 m
1 inch (in) = 0.0254 m
1 yard (yd) = 0.9144 m
1 mile = 1609.344 m
Measurement of Areas:
1 acre = 4046.8564 m^2

Investigation (Formula for Area of a Parallelogram)

1. The new quadrilateral *CDEF* is a rectangle.
2. Length of CF = length of $DE = h$
Length of EF = length of EB + length of BF
= length of EB + length of AE
$= b$
3. Area of parallelogram $ABCD$ = area of rectangle $CDEF$
$= EF \times CF$
$= bh$
4. Divide the parallelogram $ABCD$ into two triangles ABC and ADC by drawing the diagonal AC as shown below:



Length of CF = length of $DE = h$

Area of parallelogram $ABCD$ = area of $\triangle ABC$ + area of $\triangle ADC$

$= \frac{1}{2} \times AB \times CF + \frac{1}{2} \times DC \times DE$

$= \frac{1}{2}bh + \frac{1}{2}bh$

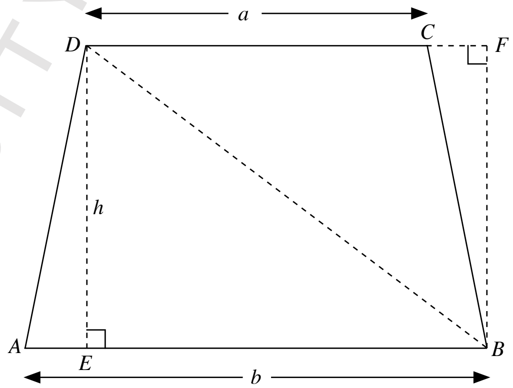
$= bh$

Thinking Time (Page 140)

From the geometry software template 'Area of Parallelogram', we can conclude that the formula for the area of parallelogram is also applicable to oblique parallelograms.

Investigation (Formula for Area of a Trapezium)

1. The new quadrilateral *AFGD* is a parallelogram.
2. Length of AF = length of AB + length of EF
$= b + a$
$= a + b$
3. Area of trapezium $ABCD = \frac{1}{2} \times$ area of parallelogram $AFGD$
$= \frac{1}{2} \times AF \times h$
$= \frac{1}{2}(a + b)h$
4. **Method 1:**
Divide the trapezium $ABCD$ into two triangles ABD and DCB by drawing the diagonal BD as shown below:



Length of FB = length of $DE = h$

Area of trapezium $ABCD$ = area of $\triangle ABD$ + area of $\triangle DCB$

$= \frac{1}{2} \times AB \times DE + \frac{1}{2} \times DC \times FB$

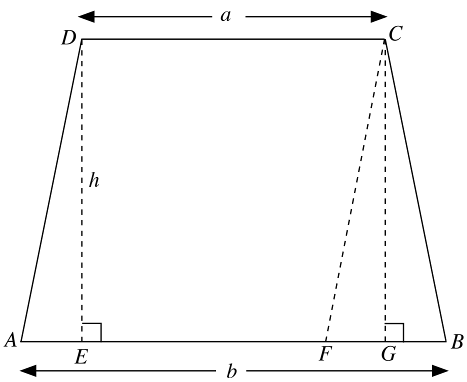
$= \frac{1}{2} \times b \times h + \frac{1}{2} \times a \times h$

$= \frac{1}{2}(b + a)h$

$= \frac{1}{2}(a + b)h$

Method 2:

Divide the trapezium $ABCD$ into a parallelogram $AFCD$ and a triangle FBC by drawing a line FC // AD as shown below:



Length of CG = length of $DE = h$

Length of AF = length of $DC = a$

$\therefore$ Length of FB = length of AB – length of AF

$= b - a$

Area of trapezium $ABCD$

= area of parallelogram $AFCD$ + area of $\triangle FBC$

$= AF \times DE + \frac{1}{2} \times FB \times CG$

$= a \times h + \frac{1}{2} \times (b - a) \times h$

$= \frac{1}{2}(2a + b - a)h$

$= \frac{1}{2}(a + b)h$

Teachers may wish to get higher-ability students to come up with more methods to find a formula for the area of a trapezium.

Thinking Time (Page 144)

1. **(i)** The new figure is a parallelogram.

(ii) Area of trapezium $= \frac{1}{2}(a + b)h$

When $a = b$,

$\frac{1}{2}(a + b)h = \frac{1}{2}(b + b)h$

$= \frac{1}{2}(2b)h$

$= bh$

= area of parallelogram

2. **(i)** The new figure is a triangle.

(ii) Area of trapezium $= \frac{1}{2}(a + b)h$

When $a = 0$,

$\frac{1}{2}(a + b)h = \frac{1}{2}(0 + b)h$

$= \frac{1}{2}bh$

= area of triangle

Practise Now 1

(a) $16 \text{ m}^2 = 16 \times 10\ 000 \text{ cm}^2$

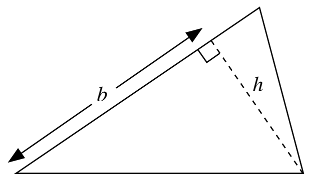
$= 160\ 000 \text{ cm}^2$

(b) $357 \text{ cm}^2 = 357 \times 0.0001 \text{ m}^2$

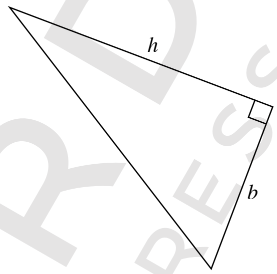
$= 0.0357 \text{ m}^2$

Practise Now 2

(a)



(b)



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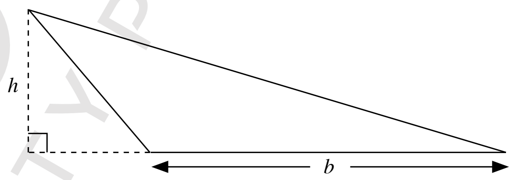
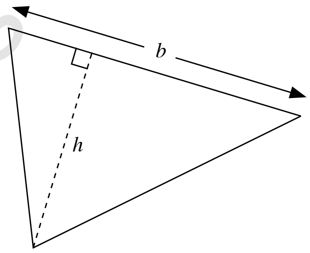
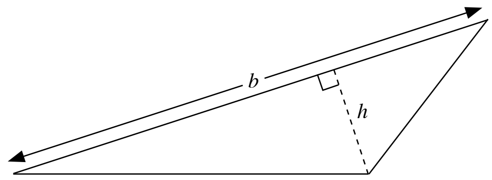


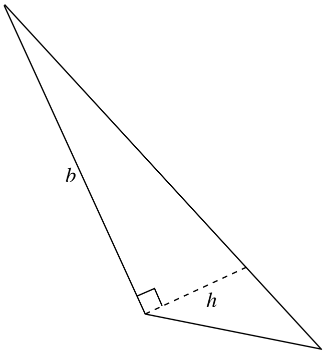
(d)



(e)



(f)



Practise Now 3

1. Length of each side of square field $= \dfrac{64}{4}$

$= 16$ m

Area of field $= 16^2$

$= 256 \text{ m}^2$

Area of path $= (16 + 3.5 + 3.5)^2 - 256$

$= 23^2 - 256$

$= 529 - 256$

$= 273 \text{ m}^2$

2. Area of shaded region

= area of rectangle $ABCD$ – area of $\triangle ARQ$ – area of $\triangle BRS$ – area of $\triangle CPS$ – area of $\triangle DPQ$

$= 25 \times 17 - \dfrac{1}{2} \times (25 - 14) \times 5 - \dfrac{1}{2} \times 14 \times 3 - \dfrac{1}{2} \times (25 - 8) \times (17 - 3) - \dfrac{1}{2} \times (17 - 5) \times 8$

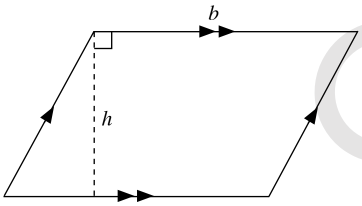
$= 425 - \dfrac{1}{2} \times 11 \times 5 - 21 - \dfrac{1}{2} \times 17 \times 14 - \dfrac{1}{2} \times 12 \times 8$

$= 425 - 27\dfrac{1}{2} - 21 - 119 - 48$

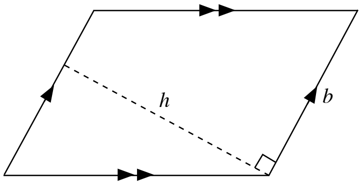
$= 209\dfrac{1}{2} \text{ m}^2$

Practise Now 4

(a)



(b)



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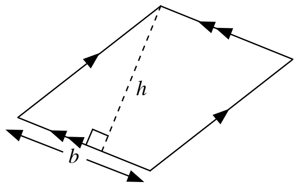

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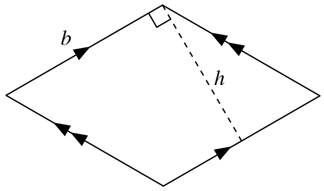

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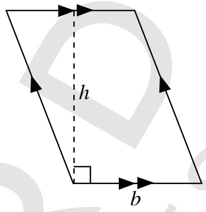

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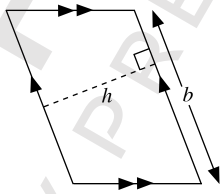


Practise Now 5

(i) Area of parallelogram $= 24 \times 7$

$= 168 \text{ m}^2$

(ii) Perimeter of parallelogram $= 2(30 + 7)$

$= 2(37)$

$= 74$ m

Practise Now 6

Area of parallelogram $= PQ \times ST = 480 \text{ m}^2$

$20 \times ST = 480$

$ST = 24$

Length of $ST = 24$ m

Practise Now 7

1. Total area of shaded regions

= area of parallelogram $ABJK$ + area of parallelogram $CDIJ$ + area of parallelogram $DEGH$

$= 4 \times 12 + (2 \times 4) \times 12 + 4 \times 12$

$= 48 + 8 \times 12 + 48$

$= 48 + 96 + 48$

$= 192 \text{ m}^2$

2. Area of $\triangle CDF = \frac{1}{2} \times DC \times CF = 60 \text{ cm}^2$

$$\frac{1}{2} \times DC \times 3CG = 60$$

$$\frac{3}{2} \times DC \times CG = 60$$

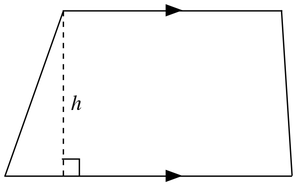
$$DC \times CG = 40$$

Area of parallelogram $ABCD = DC \times CG$

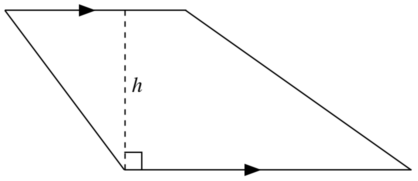
$= 40 \text{ cm}^2$

Practise Now 8

(a)



(b)



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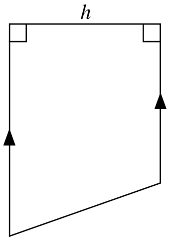


Practise Now 9

(i) Area of trapezium $= \frac{1}{2} \times (5 + 13.2) \times 4$

$= \frac{1}{2} \times 18.2 \times 4$

$= 36.4 \text{ m}^2$

(ii) Perimeter of trapezium $= 5 + 6 + 13.2 + 5.5$

$= 29.7$ m

Practise Now 10

(i) Area of trapezium $= \frac{1}{2} \times (PQ + RS) \times PS = 72 \text{ m}^2$

$$\frac{1}{2} \times (14 + 10) \times PS = 72$$

$$\frac{1}{2} \times 24 \times PS = 72$$

$$12 \times PS = 72$$

$$PS = 6$$

Length of $PS = 6$ m

(ii) Perimeter of trapezium $= PQ + QR + RS + PS = 37.2$ m

$$14 + QR + 10 + 6 = 37.2$$

$$30 + QR = 37.2$$

$$QR = 7.2$$

Length of $QR = 7.2$ m

Exercise 8A

1. **(a)** $40 \text{ m}^2 = 40 \times 10\,000 \text{ cm}^2$

$= 400\,000 \text{ cm}^2$

(b) $16 \text{ cm}^2 = 16 \times 0.0001 \text{ m}^2$

$= 0.0016 \text{ m}^2$

(c) $0.03 \text{ m}^2 = 0.03 \times 10\,000 \text{ cm}^2$

$= 300 \text{ cm}^2$

(d) $28\,000 \text{ cm}^2 = 28\,000 \times 0.0001 \text{ m}^2$

$= 2.8 \text{ m}^2$

2. **(i)** Breadth of rectangle $= \frac{259}{18.5}$

$= 14$ cm

(ii) Perimeter of rectangle $= 2(18.5 + 14)$

$= 2(32.5)$

$= 65$ cm

3. Area of figure = area of square – area of triangle

$= 9^2 - \frac{1}{2} \times 3 \times 2.5$

$= 81 - 3.75$

$= 77.25 \text{ m}^2$

4. Perimeter of unshaded region $= 2\,(l + b)$

$= 2\,(21 + 14)$

$= 2\,(35)$

$= 70$ cm

Area of unshaded region

= Area of whole rectangle – area of shaded rectangle

$= 21 \times 16 - 21 \times 2$

$= 336 - 42$

$= 294 \text{ cm}^2$

5. **(i)** Perimeter of figure $= 2\pi(2) + 2(9 - 2 \times 2) + 2(3)$

$= 4\pi + 2(5) + 6$

$= 4\pi + 10 + 6$

$= 4\pi + 16$

$= 28.6$ m (to 3 s.f.)

(ii) Area of figure = area of rectangle – area of four quadrants

$= 9 \times [2(2) + 3] - \pi(2)^2$

$= 9 \times 7 - 4\pi$

$= 63 - 4\pi$

$= 50.4 \text{ m}^2$ (to 3 s.f.)

6. Let the breadth of the rectangular field be x m.

Then the length of the field is $(x + 15)$ m.

$$2[(x + 15) + x] = 70$$
$$2(2x + 15) = 70$$
$$2x + 15 = 35$$
$$2x = 20$$
$$x = 10$$

$\therefore$ Breadth of field = 10 m

Length of field = 10 + 15
= 25 m

Area of field = 25 × 10
= 250 m^2

Area of path = (25 + 2.5 + 2.5) × (10 + 5 + 5) – 250
= 30 × 20 – 250
= 600 – 250
= 350 m^2

7. Area of shaded region = area of $\triangle ABC$ – area of $\triangle ADE$

$$= \frac{1}{2} \times 20 \times 21 - \frac{1}{2} \times 10 \times 10.5$$
$$= 210 - 52.5$$
$$= 157.5 \text{ m}^2$$

8. Area of triangle = 340 cm^2

$$\frac{1}{2} \times AC \times BD = 340$$
$$\frac{1}{\cancel{2}} \times \cancelto{10}{20} \times BD = 340$$
$$BD = \frac{340}{10}$$
$$BD = 34 \text{ cm}$$

Exercise 8B

1. **(a)** Area of parallelogram = 12 × 7
= 84 cm^2

(b) Base of parallelogram = $\frac{42}{6}$
= 7 m

(c) Height of parallelogram = $\frac{42.9}{7.8}$
= 5.5 mm

2. **(a)** Area of trapezium = $\frac{1}{2} \times (7 + 11) \times 6$
$= \frac{1}{2} \times 18 \times 6$
= 54 cm^2

(b) Height of trapezium = $\dfrac{126}{\frac{1}{2} \times (8 + 10)}$
$= \dfrac{126}{\frac{1}{2} \times 18}$
$= \dfrac{126}{9}$
= 14 m

(c) Length of parallel side 2 of trapezium = $\dfrac{72}{\frac{1}{2} \times 8} - 5$
$= \dfrac{72}{4} - 5$
= 18 – 5
= 13 mm

3. **(i)** Area of parallelogram = 6 × 9
= 54 cm^2

(ii) Perimeter of parallelogram = 2(10 + 6)
= 2(16)
= 32 cm

4. Area of parallelogram = $PQ \times ST = QR \times SU$

$$PQ \times 8 = 10 \times 11.2$$
$$PQ \times 8 = 112$$
$$PQ = 14$$

Length of PQ = 14 m

5. **(i)** Area of trapezium = $\frac{1}{2} \times (35.5 + 20) \times 15$
$= \frac{1}{2} \times 55.5 \times 15$
= 416.25 cm^2

(ii) Perimeter of trapezium = 35.5 + 18 + 20 + 16
= 89.5 cm

6. **(i)** Area of trapezium = $\frac{1}{2} \times (PQ + RS) \times PT = 150 \text{ m}^2$

$$\frac{1}{2} \times (12 + RS) \times 10 = 150$$
$$5 \times (12 + RS) = 150$$
$$12 + RS = 30$$
$$RS = 18$$

Length of RS = 18 m

(ii) Perimeter of trapezium = $PQ + QR + RS + PS$ = 54.7 m

$$12 + QR + 18 + 13 = 54.7$$
$$43 + QR = 54.7$$
$$QR = 11.7$$

Length of QR = 11.7 m

7. Area of shaded regions = area of trapezium $ABCD$ – area of $\triangle BCE$

$$= \frac{1}{2} \times (10 + 14) \times 12 - \frac{1}{2} \times 14 \times 12$$
$$= \frac{1}{2} \times 24 \times 12 - 84$$
$$= 144 - 84$$
$$= 60 \text{ cm}^2$$

8. Area of jogging trade = Area of whole parallelogram – Area of inner parallelogram

= 140 × 40 – 136 × 37
= 5600 – 5032
= 568 m^2

9. Area of $\triangle AED = \frac{1}{2} \times AE \times ED = 25 \text{ cm}^2$

$AE \times ED = 50$

Area of trapezium $BCDE = \frac{1}{2} \times (EB + DC) \times ED$

$= \frac{1}{2} \times (3AE + 4AE) \times ED$

$= \frac{1}{2} \times 7AE \times ED$

$= \frac{7}{2} \times AE \times ED$

$= \frac{7}{2} \times 50$

$= 175 \text{ cm}^2$

10. Area of triangle $= 80\text{cm}^2$

$\frac{1}{2} \times FG \times EG = 80$

$\frac{1}{2} \times 10 \times EG = 80$

$EG = \frac{80}{5}$

$EG = 16$ cm

Since $DE = EC = EG$

hence,

$DE = EC = 16$ cm

and

$DC = DE + EC$

$= 16 + 16$

$= 32$ cm

Area of parallelogram $= b \times h$

$= AB \times EG$

$= 32 \times 16$

$= 512 \text{ cm}^2$

Review Exercise 8

1. **(a)** Area of shaded region

$= 11 \times 13 + 7 \times (14 + 13) + 8 \times (35 - 20) + 9 \times 35 - 12 \times 9$

$= 143 + 7 \times 27 + 8 \times 15 + 315 - 108$

$= 143 + 189 + 120 + 315 - 108$

$= 659 \text{ cm}^2$

(b) Area of shaded region

= Area of bigger triangle – Area of smaller triangle.

$= \frac{1}{2} \times (16+48) \times 20 - \frac{1}{2} \times (10 + 42) \times 14$

$= 42 - 17.5$

$= 24.5 \text{ cm}^2$

(c) Total area of shaded regions

$= \frac{1}{2} \times (48 + 16) \times 20 + \frac{1}{2} \times (30 + 20) \times 16$

$= \frac{1}{2} \times 64 \times 20 + \frac{1}{2} \times 50 \times 16$

$= 640 + 400$

$= 1040 \text{ cm}^2$

(d) Area of shaded region

= Area of bigger trapezium – Area of smaller trapezium.

$= \frac{1}{2} \times (16 + 48) \times 20 - \frac{1}{2} \times (20 + 42) \times 14$

$= 64 \times 10 - 25 \times 7$

$= 640 - 364$

$= 276 \text{ cm}^2$

2. **(i)** Area of parallelogram $= 9 \times 25$

$= 225 \text{ m}^2$

(ii) Perimeter of parallelogram $= 2(9 + 30.8)$

$= 2(39.8)$

$= 79.6$ m

3. Area of border

= Area of Bigger rectangle – Area of smaller rectangle

$= 11 \times 15 - 5 \times 12$

$= 165 - 60$

$= 105 \text{ cm}^2$

4. Let $AB = BC = CD = DE = EF = AF = x$ cm.

$(x + x) \times x = 24$

$2x \times x = 24$

$2x^2 = 24$

$x^2 = 12$

Since $x > 0$, $x = \sqrt{12}$

Area of parallelogram $BCEF = \sqrt{12} \times \sqrt{12}$

$= 12 \text{ cm}^2$

5. Area of trapezium $ABPQ = \frac{1}{2} \times (8 + 8 \div 2) \times (6 \div 2)$

$= \frac{1}{2} \times (8 + 4) \times 3$

$= \frac{1}{2} \times 12 \times 3$

$= 18 \text{ cm}^2$

6. Area of figure

= area of rectangle $ABCF$ + area of trapezium $FCDE$

$= 20 \times 15 + \frac{1}{2} \times (20 + 3.5) \times 7$

$= 300 + \frac{1}{2} \times 23.5 \times 7$

$= 300 + 82.25$

$= 382.25 \text{ m}^2$

$= 382.25 \times 0.0001$ ha

$= 0.038\ 225$ ha

7. Area of trapezium $= 36 \text{ cm}^2$

$\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{h} = 36 \text{ cm}^2$

$\frac{1}{2} \times (\text{sum of parallel sides}) \times 6 = 36$

Sum of parallel sides $= 36\ 3$

$= 108$ cm

Challenge Yourself

1. Let the length of AB be x cm.

Then the length of BC = the length of $AC = 2x$ cm.

Area of $\triangle ABC$

= area of $\triangle ABD$ + area of $\triangle BCD$ + area of $\triangle ACD$

$= \frac{1}{2} \times x \times 9 + \frac{1}{2} \times 2x \times 7 + \frac{1}{2} \times 2x \times 7$

$= 4.5x + 7x + 7x$

$= 18.5x$ cm^2

Case 1: The base of $\triangle ABC$ is taken to be AB.

$$\frac{1}{2} \times x \times h_1 = 18.5x$$

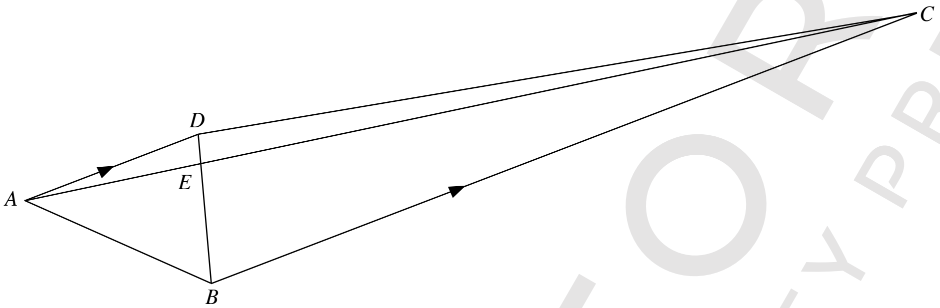
$$\therefore h_1 = 37 \text{ cm}$$

Case 2: The base of $\triangle ABC$ is taken to be BC or AC.

$$\frac{1}{2} \times 2x \times h_2 = 18.5x$$

$$\therefore h_2 = 18.5 \text{ cm}$$

2.



$\frac{BD}{BE} = \frac{5}{4} \Leftrightarrow \frac{ED}{BE} = \frac{1}{4}$

$\frac{\text{Area of } \triangle AED}{\text{Area of } \triangle ABE} = \frac{ED}{BE} = \frac{1}{4}$

$\frac{\text{Area of } \triangle AED}{20} = \frac{1}{4}$

$\therefore$ Area of $\triangle AED = 5$ cm^2

Since $\triangle ACD$ shares the same base AD and the same height as $\triangle ABD$,

area of $\triangle ACD$ = area of $\triangle ABD$.

Since $\triangle AED$ is a common part of $\triangle ACD$ and $\triangle ABD$,

area of $\triangle DCE$ = area of $\triangle ABE = 20$ cm^2.

$\frac{\text{Area of } \triangle BCE}{\text{Area of } \triangle DCE} = \frac{BE}{ED} = \frac{4}{1}$

$\frac{\text{Area of } \triangle BCE}{20} = \frac{4}{1}$

$\therefore$ Area of $\triangle BCE = 80$ cm^2

Area of trapezium

= area of $\triangle ABE$ + area $\triangle AED$ + area of $\triangle DCE$ + area of $\triangle BCE$

= 20 + 5 + 20 + 80

= 125 cm^2

Chapter 9 Volume and Surface Area of Cubes and Cuboids

TEACHING NOTES

Suggested Approach

Students have learnt the conversion of unit area and perimeter and area of plane figures in the last chapter. This chapter will be dealing with the conversion of unit volumes and the volume and surface area of solids, which is a natural transition from the last chapter, from two-dimensional to three-dimensional. To assist in the students' understanding, teachers should continually remind students to be aware of the linkages between both topics, as well as introducing real-life applications that can reinforce learning.

Section 9.1: Conversion of Units

Teachers should recap the unit conversion of lengths and areas, proceed to introduce of volume by stating actual applications (see Class Discussion: Measurement in Daily Lives), and then stating the different units associated with volume (e.g. m, cm^3 and m^3).

Students should recognise how the number of dimensions and the unit representation for lengths, areas and volumes are related (e.g. cm, cm^2 and cm^3). Students should recall calculations such as $1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$ and solve problems involving conversion of unit volumes.

Section 9.2: Recognise and Identify 3D Shapes and their Properties

Teachers should first define and explain that nets are basically flattened figures that can be folded to its three-dimensional solids. The teacher should revise the different properties of three dimensional solids.

Teachers should show the nets of the various solids. Students are encouraged to make their own nets and form the different three-dimensional solids. They should also be able to visualise the solids from different viewpoints.

Section 9.3: Volume and Surface Area of Cubes and Cuboids

Teachers can state that the volume of an object refers to the space it occupies, so the greater the volume, the more space the object occupies.

Students should be informed and know that the volume of cubes and cuboids is the product of its three sides (base × height = (length × breadth) × height).

The formulas for the total surface area of cubes and cuboids can be explored and discovered by students (see Class Discussion: Surface Area of Cubes and Cuboids). It is important for the students to observe that the total surface area is the total area of all its faces.

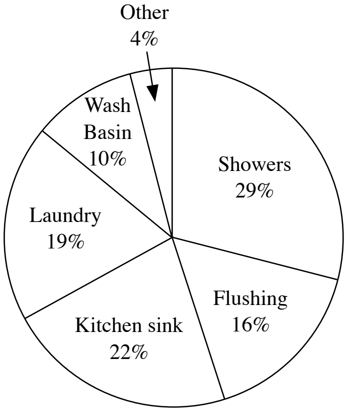
Challenge Yourself

Teachers should challenge students to think how the cross-section of the cuboid looks like in finding the volume and surface area.

WORKED SOLUTIONS

Class Discussion (Measurements in Daily Lives)

1. **(i)**



Source:
http://www.pub.gov.sg/conserve/Households/Pages/Watersavinghabits.aspx

The activity which requires the greatest amount of water is shower.

(ii) –

Some measures:

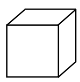
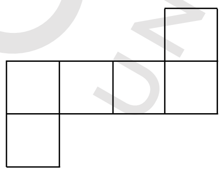
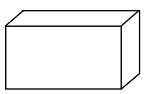
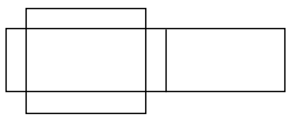
- Take shorter showers.
- Turn off the shower tap while soaping.
- Use a tumbler when brushing your teeth.
- Do not thaw food under running water. Let it defrost overnight inside the refrigerator instead.
- Wash vegetables and dishes in a sink or container filled with water.
- Install thimbles or water saving devices at taps with high flow rate.
- Turn off taps tightly to ensure they do not drip.
- Do not leave the tap running when not in use.

2. **(i)** The volume of one teaspoon of liquid is 5 m*l*.

(ii) This corresponds to 2 litres of water.

Investigation (Cubes, Cuboids Prisms and Cylinders)

Part II:

Name	Figure	Net
Cube		
Cuboid		

Class Discussion (Surface Area of Cubes and Cuboids)

1. A cube has <u>6</u> surfaces. Each surface is in the shape of a <u>square</u>.
The area of each face is <u>equal</u>.
$\therefore$ The total surface area of a cube is $6l^2$.
A cuboid has <u>6</u> surfaces. Each surface is in the shape of a <u>rectangle</u>.
$\therefore$ The total surface area of a cube is $2(b \times l + b \times h + l \times h)$.

2. The total surface of the object is equal to the total area of all the faces of the net.

Practise Now 1

(a) **(i)** $1 \text{ m}^3 = 1\ 000\ 000 \text{ cm}^3$

$10 \text{ m}^3 = 10 \times 1\ 000\ 000 \text{ cm}^3$

$= 10\ 000\ 000 \text{ cm}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ m}l$

$10\ 000\ 000 \text{ cm}^3 = 10\ 000\ 000 \text{ m}l$

$10 \text{ m}^3 = 10\ 000\ 000 \text{ m}l$

(b) **(i)** $1\ 000\ 000 \text{ cm}^3 = 1 \text{ m}^3$

$$165\ 000 \text{ cm}^3 = \frac{165\ 000}{1\ 000\ 000} \text{ m}^3$$

$= 0.165 \text{ m}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ m}l$

$165\ 000 \text{ cm}^3 = 165\ 000 \text{ m}l$

$$= \frac{165\ 000}{1000}\ l$$

$= 165\ l$

Practise Now 2

1. **(i)** Volume of the cuboid $= l \times 18 \times 38 = 35\ 568$

$$l = \frac{35\ 568}{18 \times 38}$$

$= 52$

(ii) Volume of each small cube $= 2 \times 2 \times 2 = 8 \text{ cm}^3$

Number of cubes to be obtained

$$= \frac{35\ 568}{8}$$

$= 4446$

2. Volume of the open rectangular tank

$= 55 \times 35 \times 36$

$= 69\ 300 \text{ cm}^3$

Volume of water in the open rectangular tank initally

$$= \frac{1}{2} \times 69\ 300$$

$= 34\ 650 \text{ cm}^3$

Total volume of water in the open rectangular tank after 7700 cm^3 of water are added to it

$= 34\ 650 + 7700$

$= 42\ 350 \text{ cm}^3$

Let the depth of water in the tank be d cm.

$55 \times 35 \times d = 42\ 350$

$1925d = 42\ 350$

$d = 22$

Depth of water = 22 cm

Practise Now 3

Volume of cube $= l^3$
$= 80^3$
$= 512000 \text{ cm}^3$

Practise Now 4

1. **(i)** Volume of cuboid $= 8 \times 5 \times 10$
$= 400 \text{ cm}^3$

(ii) Surface area of the cuboid $= 2(8 \times 5 + 8 \times 10 + 5 \times 10)$
$= 340 \text{ cm}^2$

2. **(i)** Volume of water in the tank
$= 16 \times 9 \times 8$
$= 1152 \text{ cm}^3$
$= 1152 \text{ m}l$
$= \frac{1152}{1000}\ l$
$= 1.152\ l$

(ii) Surface area of the tank that is in contact with the water
$= (16 \times 9) + 2(16 \times 8 + 9 \times 8)$
$= 544 \text{ cm}^2$

3. Let the length of the cube be l cm.
$l \times l \times l = 27 \text{ cm}^3$
$l^3 = 27$
$l = \sqrt[3]{27}$
$l = 3$
Total area of the faces that will be coated with paint
$= 6(3 \times 3)$
$= 54 \text{ cm}^2$

Exercise 9A

1. **(a)** **(i)** $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$
$4 \text{ m}^3 = 4 \times 1\,000\,000 \text{ cm}^3$
$= 4\,000\,000 \text{ cm}^3$

(ii) $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$
$0.5 \text{ m}^3 = 0.5 \times 1\,000\,000 \text{ cm}^3$
$= 500\,000 \text{ cm}^3$

(b) **(i)** $1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$
$250\,000 \text{ cm}^3 = \frac{250\,000}{1\,000\,000} \text{ m}^3$
$= 0.25 \text{ m}^3$

(ii) $1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$
$67\,800 \text{ cm}^3 = \frac{67\,800}{1\,000\,000} \text{ m}^3$
$= 0.0678 \text{ m}^3$

2. **(a)** **(i)** $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$
$0.84 \text{ m}^3 = 0.84 \times 1\,000\,000 \text{ cm}^3$
$= 840\,000 \text{ cm}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ m}l$
$840\,000 \text{ cm}^3 = 840\,000 \text{ m}l$

(b) **(i)** $1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$
$2560 \text{ cm}^3 = \frac{2560}{1\,000\,000} \text{ m}^3$
$= 0.002\,56 \text{ m}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ m}l$
$2560 \text{ cm}^3 = 2560 \text{ m}l$
$= \frac{2560}{1000}\ l$
$= 2.56\ l$

3. **(a)** **(i)** Volume of the cuboid $= 6 \times 8 \times 10$
$= 480 \text{ cm}^3$

(ii) Surface area of the cuboid $= 2(6 \times 8 + 8 \times 10 + 6 \times 10)$
$= 376 \text{ cm}^2$

(b) **(i)** Volume of the cube $= l^3$
$= 7^3$
$= 343 \text{ cm}^3$

(ii) Total surface area $= 6l^2$
$= 6 \times 7^2$
$= 6 \times 49$
$= 294 \text{ cm}^2$

(c) **(i)** Volume of the cuboid $= 120 \times 10 \times 96$
$= 115\,200 \text{ mm}^3$

(ii) Surface area of the cuboid
$= 2(120 \times 10 + 96 \times 10 + 120 \times 96)$
$= 27\,360 \text{ mm}^2$

(d) **(i)** Volume $= l^3$
$= 5.8 \times 5.8 \times 5.8 \text{ cm}^3$
$= 195.11 \text{ cm}^3$

(ii) Total surface area $= 6l^2$
$= 6 \times 5.8^2 \text{ cm}^2$
$= 201.8 \simeq 202 \text{ cm}^2$

(e) **(i)** Volume of the cuboid $= 1\frac{2}{5} \times \frac{3}{8} \times \frac{5}{8}$
$= \frac{21}{64} \text{ cm}^3$

(ii) Surface area of the cuboid
$= 2\ \ 1\frac{2}{5} \times \frac{3}{8} + \frac{3}{8} \times \frac{5}{8} + 1\frac{2}{5} \times \frac{5}{8}$
$= 3\frac{43}{160} \text{ cm}^2$

(f) **(i)** Volume $= l^3$
$= (1\frac{1}{2})^3 \text{ cm}^3$
$= (\frac{3}{2})^3 \text{ cm}^3$
$= (\frac{27}{2})^3 \text{ cm}^3$
$= 3\frac{3}{8} \text{ cm}^3$

(ii) Total surface area $= 6l^2$
$= 6 \times \frac{1}{2^2} \text{ cm}^2$
$= -\frac{1}{2} \text{ cm}^2$
$= 1\frac{1}{2} \text{ cm}^2$

4.

	Length	Breadth	Height	Volume	Total surface area
(a)	24 mm	18 mm	5 mm	2160 mm^3	1284 mm^2
(b)	5 cm	3 cm	8 cm	120 cm^3	158 cm^2
(c)	2.5 cm	6 cm	3.5 cm	52.5 cm^3	89.5 cm^2
(d)	12 m	8 m	6 m	576 m^3	432 m^2

(a) Volume $= 24 \times 18 \times 5$
$= 2160$ mm^3
Surface area $= 2(24 \times 18 + 24 \times 5 + 18 \times 5)$
$= 1284$ mm^2

(b) Let the height of the cuboid be h cm.
Volume $= 5 \times 3 \times h = 120$ cm^3
$\therefore h = \dfrac{120}{5 \times 3} = 8$ cm
Surface area $= 2(5 \times 3 \times 5 \times 8 + 3 \times 8)$
$= 158$ cm^2

(c) Let the length of the cuboid be l cm.
Volume $= l \times 6 \times 3.5 = 52.5$ cm^3
$\therefore l = \dfrac{52.5}{6 \times 3.5} = 2.5$ cm
Surface area $= 2(2.5 \times 6 + 6 \times 3.5 + 2.5 \times 3.5)$
$= 89.5$ cm^2

(d) Let the breadth of the cuboid be b m.
Volume $= 12 \times b \times 6 = 576$ m^3
$\therefore b = \dfrac{576}{12 \times 6} = 8$ m
Surface area $= 2(12 \times 8 + 6 \times 8 + 12 \times 6)$
$= 432$ m^2

5. **(i)** Volume of the cuboid $= 28 \times b \times 15 = 6720$ cm^3
$\therefore b = \dfrac{6720}{28 \times 15}$
$= 16$
$\therefore$ Breadth = 16 cm

(ii) Volume of each small cube $= 4 \times 4 \times 4 = 64$ cm^3
Number of cubes to be obtained
$= \dfrac{6720}{64}$
$= 105$

6. Volume of the rectangular block of metal
$= 0.24 \times 0.19 \times 0.15$
$= 0.00684$ m^3
Let the length of the cube be l cm.
Volume of each small cube $= l \times l \times l = 0.00684$ m^3
$l^3 = 0.00684$
$l = \sqrt[3]{0.00684}$
$= 0.190$ (to 3 s.f.)
$\therefore$ Length of each side = 0.190 m

7. Volume of the open rectangular tank
$= 4 \times 2 \times 4.8$
$= 38.4$ m^3
Volume of water in the open rectangular tank initally
$= \dfrac{3}{4} \times 38.4$
$= 28.8$ m^3
4000 l = 4000 × 1000 ml
= 4 000 000 ml
= 4 000 000 cm^3
$= \dfrac{4\,000\,000}{1\,000\,000}$ m^3
$= 4$ m^3
Total volume of water in the open rectangular tank after 4000 litres of water are added to it
$= 28.8 + 4$
$= 32.8$ m^3
Let the depth of water in the tank be d m.
$4 \times 2 \times d = 32.8$
$8d = 32.8$
$d = 4.1$
$\therefore$ Depth = 4.1 m

8. External volume $= (3.2 + 0.2 + 0.2) \times (2.2 + 0.2 + 0.2) \times (1.5 + 0.2)$
$= 3.6 \times 2.6 \times 1.7$
$= 15.912$ m^3
Internal volume $= 3.2 \times 2.2 \times 1.5$
$= 10.56$ m^3
Volume of wood used $= 15.912 - 10.56$
$= 5.352$ m^3

9. External volume $= 15 \times 10 \times 45$
$= 6750$ cm^3
Internal volume $= 3 \times 2 \times 45$
$= 270$ cm^3
Volume of the hollow glass structure $= 6750 - 270$
$= 6480$ cm^3

10. **(i)** Volume of water in the tank
$= 0.2 \times 0.15 \times 0.16$
$= 0.0048$ m^3
$= 0.0048 \times 1\,000\,000$ cm^3
$= 4800$ cm^3
= 4800 ml
$= \dfrac{4800}{1000}$ l
= 4.8 l

(ii) Surface area of the tank that is in contact with the water
$= (0.2 \times 0.15) + 2(0.2 \times 0.16 + 0.15 \times 0.16)$
$= 0.142$ m^2

11. **(i)** Volume of water in the tank
$= 80 \times 40 \times 35$
$= 112\,000$ cm^3
= 112 000 ml
$= \dfrac{112\,000}{1000}$ l
= 112 l

(ii) Surface area of the tank that is in contact with the water
$= (80 \times 40) + 2(80 \times 35 + 40 \times 35)$
$= 11\ 600\ cm^2$
$= \frac{11\ 600}{10\ 000}\ m^2$
$= 1.16\ m^2$

12. Let the length of the cube be l cm.
$l \times l \times l = 64\ cm^3$
$l^3 = 64$
$l = \sqrt[3]{64}$
$= 4$
Total area of the faces that will be coated with paint
$= 6(4 \times 4)$
$= 96\ cm^2$

13. Let the length of the cube be l cm.
$6(l \times l) = 433.5$
$6l^2 = 433.5$
$l^2 = 72.25$
$l = \sqrt{72.25}$
$= 8.5$
Volume of the cube
$= 8.5 \times 8.5 \times 8.5$
$= 614.125\ cm^3$

14. Volume of wood used to make this trough
$= (185 \times 45 \times 28) - [(185 - 2.5 - 2.5) \times (45 - 2.5 - 2.5) \times (28 - 2.5)]$
$= (185 \times 45 \times 28) - (180 \times 40 \times 25.5)$
$= 233\ 100 - 183\ 600$
$= 49\ 500\ cm^3$
$= \frac{49\ 500}{1\ 000\ 000}\ m^3$
$= 0.0495\ m^3$

15. In one minute, the water will flow through $22 \times 60 = 1320$ cm along the drain.
Amount of water that will flow through in one minute
$= 30 \times 3.5 \times 1320$
$= 138\ 600\ cm^3$
$= 138\ 600\ ml$
$= \frac{138\ 600}{1000}\ l$
$= 138.6\ l$

16. **(i)** Let the height of the cuboid be h cm.
Surface area of the cuboid $= 2(12 \times 9 + 12 \times h + 9 \times h)$
$= 426\ cm^2$
$2(108 + 12h + 9h) = 426$
$2(108 + 21h) = 426$
$108 + 21h = 213$
$21h = 213 - 108$
$21h = 105$
$h = 5$
$\therefore$ Height of cuboid = 5 cm

(ii) Volume of the cuboid
$= 12 \times 9 \times 5$
$= 540\ cm^3$

17. **(i)** Floor area of Room $A = 26 \times 1$
$= 26\ m^2$
Volume of Room $A = 26 \times 1 \times 3$
$= 78\ m^3$
Floor area of Room $B = 5 \times 5$
$= 25\ m^2$
Volume of Room $B = 5 \times 5 \times 3$
$= 75\ m^3$
Floor area of Room $C = 6 \times 6$
$= 36\ m^2$
Volume of Room $C = 6 \times 6 \times 1.8$
$= 64.8\ m^3$

(ii) No. If both rooms, A and B, have the same height, then we will use the floor area as the gauge. If the rooms do not have the same height, then we will use the volume to decide.

Review Exercise 9

1. **(a)** **(i)** Volume of cuboid
$= l \times b \times h$
$= (3 + 6 + 3) \times 3 \times 2$
$= 12 \times 6$
$= 72\ cm^2$

(ii) Total surface area
$= 2\ (lb + bh + hl)$
$= 2\ (12 \times 3 + 3 \times 2 + 12 \times 2)$
$= 2\ (36 + 6 + 24)$
$= 2\ (96)$
$= 192\ cm^2$

(b) **(i)** Volume = Volume of cuboid 1 + Volume of cuboid 2
$= 5 \times 3 \times 2 + 3 \times 4 \times 2$
$= 30 + 24$
$= 54\ cm^3$

(ii) Total surface area
$= 2 \times 5 + 7 \times 2 + 4 \times 2 + 2 \times 2 + 3 \times 2 + 3 \times 2 + 2(3 \times 4) + 2(3 \times 5)$
$= 10 + 14 + 8 + 4 + 6 + 6 + 24 + 30$
$= 102\ cm^2$

(c) **(i)** Volume of the solid
$= 4 \times 5 \times 1 - 2(1 \times 3)$
$= 20 - 6$
$= 14\ cm^3$

(ii) Total surface area of the solid
$= 2(1 \times 4) + 8(1 \times 1) + 2(1 \times 3) + 2[4 \times 5 - 2(1 \times 3)]$
$= 8 + 8 + 6 + 40 - 12$
$= 50\ cm^2$

(d) **(i)** Volume of the solid

$= 1 \times 1 \times 5 + 2 \times 4 \times 1 + 1 \times 1 \times 3$

$= 5 + 8 + 3$

$= 16 \text{ cm}^3$

(ii) Total surface area of the solid

$= 2(1 \times 5) + 2(1 \times 1) + 2(1 \times 3) + 2(1 \times 4)$

$+ 2[1 \times 5 + 2 \times 3 + 1 \times 5]$

$= 10 + 2 + 6 + 8 + 32$

$= 58 \text{ cm}^2$

2. 4.5 m = 450 cm, 3.6 m = 360 cm

Number of bricks required

$= \frac{450}{18} \times \frac{18}{9} \times \frac{360}{6}$

$= 3000$

3. Volume of the rectangular block of metal

$= 256 \times 152 \times 81$

$= 3\ 151\ 872 \text{ mm}^3$

Let the length of the cube be l mm.

$l^3 = 3\ 151\ 872$

$l = \sqrt[3]{3\,151\,872}$

$= 147$ (to 3 s.f.)

$\therefore$ Length of each side = 147 mm

4. Let the length of the cube be l cm.

$l^3 = 343$

$l = \sqrt[3]{343}$

$= 7$

Total surface area of a cube

$= 6l^2$

$= 6(7)^2$

$= 294 \text{ cm}^2$

Challenge Yourself

(i) Volume of the solid

$= 50 \times 70 \times 30 - 10 \times 10 \times 70 - 2(10 \times 10 \times 10) - 2(10 \times 10 \times 20)$

$= 105\ 000 - 7000 - 4000 - 2000$

$= 92\ 000 \text{ cm}^3$

(ii) Total surface area of the solid

$= 2(30 \times 70 - 10 \times 10) + 2(50 \times 30 - 10 \times 10) + 2(50 \times 70 - 10 \times 10)$

$+ 4(10 \times 60) + 8(10 \times 10) + 8(10 \times 20)$

$= 4000 + 2800 + 6800 + 2400 + 800 + 1600$

$= 18\ 400 \text{ cm}^2$

Chapter 10 Basic Geometry

TEACHING NOTES

Suggested Approach

Students have learnt angle measurement in primary school. They have learnt the properties, namely, angles on a straight line, angles at a point and vertically opposite angles. However, students are unfamiliar with the types of angles and using algebraic terms in basic geometry. There is a need to guide students to apply basic algebra and linear equations in this topic. Students will learn how to do this through the worked examples in this topic. Teachers can introduce basic geometry by showing real-life applications (see chapter opener on page 161).

Section 10.1: Points, Lines and Planes

Teachers should illustrate what a point, a line, intersecting lines and planes look like. Teachers can impress upon the students that there is a difference between a line and a ray. A ray has a direction while a line has no direction. Teachers can highlight to the students that for a ray, the arrowhead indicates the direction in which the ray extends while for a line, its arrowhead is to indicate that the line continues indefinitely.

The thinking time on page 164 of the textbook requires students to think and determine whether each of the statements is true or false. Teachers should make use of this opportunity to highlight and clear some common misconceptions about points, lines and planes.

Section 10.2: Angles

Teachers can build upon prerequisites, namely angle measurement, to introduce the types of angles by classifying angle measurements according to their sizes.

To make practice more interesting, teachers can get the students to work in groups to measure and classify the various types of angles of different objects (i.e. scissors, set square, compass and the hands of a clock).

Teachers should recap with students on what they have learnt in primary school, i.e. angles on a straight line, angles at a point and vertically opposite angles. After going through Worked Examples 1 to 4, students should be able to identify the properties of angles and use algebraic terms to form and solve a linear equation to find the value of the unknowns. Students are expected to state reasons in their working.

Section 10.3: Angles Formed by Two Parallel Lines and a Transversal

Teachers can get students to discuss examples where they encounter parallel lines in their daily lives and ask them what happens when a line or multiple lines cut the parallel lines.

To make learning more interactive, students are given the opportunity to explore the three angle properties observed when a pair of parallel lines is cut by a transversal (see Investigation: Corresponding Angles, Alternate Angles and Interior Angles). Through this investigation, students should be able to observe the properties of angles associated with parallel lines. The investigation also helps students to learn how to solve problems involving angles formed by two parallel lines and a transversal. Students are expected to use appropriate algebraic variables to form and solve linear equations to find the value of the unknowns. Teachers should emphasise the importance of stating the properties when the students are solving questions on basic geometry.

Challenge Yourself

Question 1: Teachers can guide the students by hinting to them that this question is similar to a problem involving number patterns. Students have to draw a table and write down the first few numbers of rays between OA and OB, and their respective number of different angles. The students will then have to observe carefully and find an expression that represents rays between OA and OB.

Question 2: Teachers can guide the students by telling them to find the different angles that both the hour hand and minute hand makes from one specific position to another.

Question 3: Teachers can guide the students by telling them to find the number of times the bell will sound between certain times of the day.

WORKED SOLUTIONS

Thinking Time (Page 164)

(a) False. There are an infinite number of points lying on a line segment.
(b) False. There is exactly one line that passes through any three distinct points which are collinear; there is no line that passes through any three distinct points which are non-collinear.
(c) False. There is exactly one line that passes through any two distinct points.
(d) False. Two distinct lines intersect at one point; two coincident lines intersect at an infinite number of points; two parallel lines do not intersect at any point.
(e) True.

Practise Now 1

(a) Acute
(b) Reflex
(c) Obtuse
(d) Obtuse
(e) Reflex
(f) Acute

Practise Now 2

1. **(a)** $122° + a° = 180°$ (adj. $\angle$s on a str. line)
$a° = 180° - 122°$
$= 58°$
$\therefore a = 58$

(b) $95° + 65° + b° = 180°$ (adj. $\angle$s on a str. line)
$b° = 180° - 95° - 65°$
$= 20°$
$\therefore b = 20$

2. $2c° + 100° + 3c° = 180°$ (adj. $\angle$s on a str. line)
$2c° + 3c° = 180° - 100°$
$5c° = 80°$
$c° = 16°$
$\therefore c = 16$

Practise Now 3

1. $58° + 148° + 7a° = 360°$ ($\angle$s at a point)
$7a° = 360° - 58° - 148°$
$= 154°$
$a° = 22°$
$\therefore a = 22$

2. $b° + 90° + b° + 4b° = 360°$ ($\angle$s at a point)
$b° + b° + 4b° = 360° - 90°$
$6b° = 270°$
$b° = 45°$
$\therefore b = 45$

Practise Now 4

(i) $A\hat{O}C + 90° + 53° = 180°$ (adj. $\angle$s on a str. line)
$A\hat{O}C = 180° - 90° - 53°$
$= 37°$

(ii) $B\hat{O}D = A\hat{O}C$
$= 37°$ (vert. opp. $\angle$s)

Practise Now 5

$3a° + 40° = a° + 60°$ (vert. opp. $\angle$s)
$3a° - a° = 60° - 40°$
$2a° = 20°$
$a° = 10°$
$\therefore a = 10$
$a° + 60° + 4b° + 10° = 180°$ (adj. $\angle$s on a str. line)
$10° + 60° + 4b° + 10° = 180°$
$4b° = 180° - 10° - 60° - 10°$
$= 100°$
$b° = 25°$
$\therefore b = 25$

Practise Now 6

(a) **(i)** $\angle a$ and $\angle m$, $\angle b$ and $\angle n$, $\angle c$ and $\angle o$, $\angle d$ and $\angle p$, $\angle e$ and $\angle i$, $\angle f$ and $\angle j$, $\angle g$ and $\angle k$, $\angle h$ and $\angle l$
(ii) $\angle c$ and $\angle m$, $\angle d$ and $\angle n$, $\angle g$ and $\angle i$, $\angle h$ and $\angle j$
(iii) $\angle c$ and $\angle n$, $\angle d$ and $\angle m$, $\angle g$ and $\angle j$, $\angle h$ and $\angle i$

(b) No, $\angle c \neq \angle g$ as PQ is not parallel to RS.

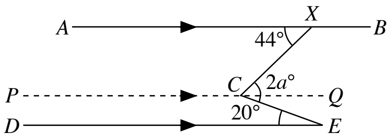
Practise Now 7

1. $a° = 54°$ (corr. $\angle$s, AB // CD)
$\therefore a = 54$
$c° + 106° = 180°$ (int. $\angle$s, AB // CD)
$c° = 180° - 106°$
$= 74°$
$\therefore c = 74$
$b° = c°$ (vert. opp. $\angle$s)
$= 74°$
$\therefore b = 74$
$d° = c°$ (corr. $\angle$s, AB // CD)
$= 74°$
$\therefore d = 74$

2. $2e° + 30° = 69°$ (corr. $\angle$s, AB // CD)
$2e° = 69° - 30°$
$= 39°$
$e° = 19.5°$
$\therefore e = 19.5$
$f° = 2e°$ (corr. $\angle$s, AB // CD)
$= 39°$
$\therefore f = 39$

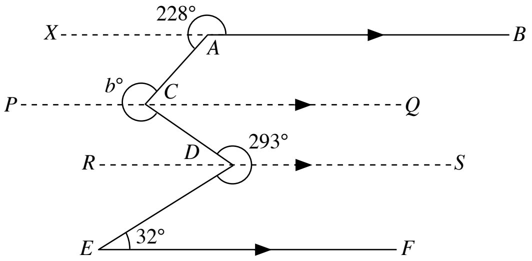
Practise Now 8

1.



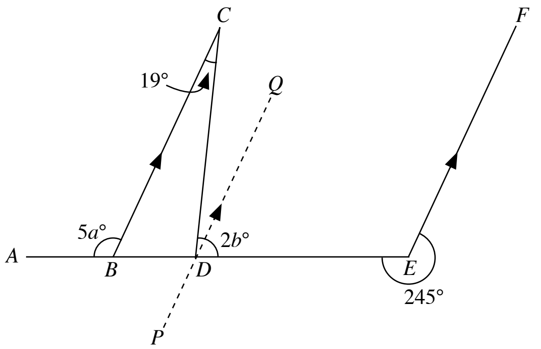
$X\hat{C}Q = 44°$ (alt. $\angle$s, AB // PQ)
$E\hat{C}Q = 20°$ (alt. $\angle$s, PQ // DE)
$2a° = X\hat{C}Q + E\hat{C}Q$
$= 44° + 20°$
$= 64°$
$a° = 32°$
$\therefore a = 32$

2.



$X\hat{A}C = 228° - 180°$
$= 48°$
$A\hat{C}Q = X\hat{A}C$ (alt. $\angle$s, XB // PQ)
$= 48°$
$E\hat{D}R = 32°$ (alt. $\angle$s, RS // EF)
$E\hat{D}R + C\hat{D}R + 293° = 360°$ ($\angle$s at a point)
$32° + C\hat{D}R + 293° = 360°$
$C\hat{D}R = 360° - 32° - 293°$
$= 35°$
$D\hat{C}Q = C\hat{D}R = 35°$ (alt. $\angle$s, PQ // RS)
$b° + A\hat{C}Q + D\hat{C}Q = 360°$ ($\angle$s at a point)
$b° + 48° + 35° = 360°$
$b° = 360° - 35° - 48°$
$= 277°$
$\therefore b = 277$

Practise Now 9



$D\hat{E}F + 245° = 360°$ ($\angle$s at a point)
$D\hat{E}F = 360° - 245°$
$= 115°$
$5a° = 115°$ (corr. $\angle$s, BC // EF)
$a° = 23°$
$\therefore a = 23$
$C\hat{D}Q = 19°$ (alt. $\angle$s, BC // PQ)
$E\hat{D}Q + D\hat{E}F = 180°$ (int. $\angle$s, PQ // EF)
$E\hat{D}Q + 115° = 180°$
$E\hat{D}Q = 180° - 115°$
$= 65°$
$2b° = C\hat{D}Q + E\hat{D}Q$
$= 19° + 65°$
$= 84°$
$b° = 42°$
$\therefore b = 42$

Practise Now 10

Since $B\hat{W}Q = D\hat{Y}Q$ (= 122°), then AB // CD (converse of corr. $\angle$s).
$\therefore B\hat{X}S = C\hat{Z}R = 65°$ (alt. $\angle$s, AB // CD)

Exercise 10A

1. **(a)** $a = 79$, $b = 106$, $c = 98$
(b) $d = 50$, $e = 228$
(c) $f = 117$, $g = 45$
(d) $h = 243$, $i = 94$, $j = 56$

2. **(a)** Obtuse
(b) Reflex
(c) Acute
(d) Reflex
(e) Acute
(f) Obtuse

3. **(a)** Complementary angle of 18° = 90° – 18°
= 72°
(b) Complementary angle of 46° = 90° – 46°
= 44°
(c) Complementary angle of 53° = 90° – 53°
= 37°
(d) Complementary angle of 64° = 90° – 64°
= 26°

4. (a) Supplementary angle of $36° = 180° – 36°$
$= 144°$

(b) Supplementary angle of $12° = 180° – 12°$
$= 168°$

(c) Supplementary angle of $102° = 180° – 102°$
$= 78°$

(d) Supplementary angle of $171° = 180° – 171°$
$= 9°$

5. (a) $a° + 33° = 180°$ (adj. ∠s on a str. line)
$a° = 180° – 33°$
$= 147°$
$\therefore a = 147$

(b) $b° + 42° + 73° = 180°$ (adj. ∠s on a str. line)
$b° = 180° – 42° – 73°$
$= 65°$
$\therefore b = 65$

(c) $4c° + 80° + c° = 180°$ (adj. ∠s on a str. line)
$4c° + c° = 180° – 80°$
$5c° = 100°$
$c° = 20°$
$\therefore c = 20$

(d) $4d° + 16° + 2d° + 14° = 180°$ (adj. ∠s on a str. line)
$4d° + 2d° = 180° – 16° – 14°$
$6d° = 150°$
$d° = 25°$
$\therefore d = 25$

6. (a) $x° + y° + z° = 180°$ (adj. ∠s on a str. line)
When $y° = 45°$, $z° = 86°$,
$x° + 45° + 86° = 180°$
$x° = 180° – 45° – 86°$
$= 49°$
$\therefore x = 49$

(b) $x° + y° + z° = 180°$ (adj. ∠s on a str. line)
When $x° = 2y°$, $z° = 3y°$,
$2y° + y° + 3y° = 180°$
$6y° = 180°$
$y° = 30°$
$\therefore y = 30$

7. (a) $a° + 67° + 52° + 135° = 360°$ (∠s at a point)
$a° = 360° – 67° – 52° – 135°$
$= 106°$
$\therefore a = 106$

(b) $5b° + 4b° + 3b° = 360°$ (∠s at a point)
$12b° = 360°$
$b° = 30°$
$\therefore b = 30$

(c) $16c° + 4c° + 90° + 4c° = 360°$ (∠s at a point)
$16c° + 4c° + 4c° = 360° – 90°$
$24c° = 270°$
$c° = 11.25°$
$\therefore c = 11.25$

(d) $(7d + 23)° + 6d° + 139° + 5d° = 360°$ (∠s at a point)
$7d° + 23° + 6d° + 139° + 5d° = 360°$
$7d° + 6d° + 5d° = 360° – 23° – 139°$
$18d° = 198°$
$d° = 11°$
$\therefore d = 11$

8. (i) $A\hat{O}C = 48°$ (vert. opp. ∠s)

(ii) $90° + D\hat{O}E + 48° = 180°$ (adj. ∠s on a str. line)
$D\hat{O}E = 180° – 90° – 48°$
$= 42°$

9. (a) $40° + 30° + a° = 117°$ (vert. opp. ∠s)
$a° = 117° – 40° – 30°$
$= 47°$
$\therefore a = 47$

(b) $7b° + 3b° = 180°$ (adj. ∠s on a str. line)
$10b° = 180°$
$b° = 18°$
$\therefore b = 18$
$c° = 7b°$ (vert. opp. ∠s)
$= 7(18°)$
$= 126°$
$\therefore c = 126$

10. (a) $x° + y° + z° = 180°$ (adj. ∠s on a str. line)
$y° + x° + z° = 180°$
When $y° = x° + z°$,
$y° + y° = 180°$
$2y° = 180°$
$y° = 90°$
$\therefore y = 90$

(b) $x° + y° + z° = 180°$ (adj. ∠s on a str. line)
When $x° = y° = z°$,
$z° + z° + z° = 180°$
$3z° = 180°$
$z° = 60°$
$\therefore z = 60$

11. $A\hat{O}B + D\hat{O}A = 180°$ (adj. ∠s on a str. line)
$A\hat{O}B + 5A\hat{O}B = 180°$
$6A\hat{O}B = 180°$
$\therefore A\hat{O}B = 30°$
$B\hat{O}C = 2A\hat{O}B$
$= 2 \times 30°$
$= 60°$
$C\hat{O}D = 4A\hat{O}B$
$= 4 \times 30°$
$= 120°$
$D\hat{O}A = 5A\hat{O}B$
$= 5 \times 30°$
$= 150°$

12. (a) $7a° + 103° = 180°$ (adj. ∠s on a str. line)

$7a° = 180° - 103°$

$= 77°$

$a° = 11°$

$\therefore a = -11$

$2b° + 13° = 103°$ (vert. opp. ∠s)

$2b° = 103° - 13°$

$= 90°$

$b° = 45°$

$\therefore b = 45$

(b) $62° + 49° + 3c° = 180°$ (adj. ∠s on a str. line)

$3c° = 180° - 62° - 49°$

$= 69°$

$c° = 23°$

$\therefore c = 23$

$d° = 3c°$ (vert. opp. ∠s)

$= 69°$

$\therefore d = 69$

$e° = 62° + 49°$ (vert. opp. ∠s)

$= 111°$

$\therefore e = 111$

(c) $7f° + 5° = 2f° + 35°$ (vert. opp. ∠s)

$7f° - 2f° = 35° - 5°$

$5f° = 30°$

$f° = 6°$

$\therefore f = 6$

$2f° + 35° + 5g° + 18° = 180°$ (adj. ∠s on a str. line)

$2(6°) + 35° + 5g° + 18° = 180°$

$12° + 35° + 5g° + 18° = 180°$

$5g° = 180° - 12° - 35° - 18°$

$= 115°$

$g° = 23°$

$\therefore g = 23$

(d) $24° + 90° + h° = 104° + 32°$ (vert. opp. ∠s)

$h° = 104° + 32° - 24° - 90°$

$= 22°$

$\therefore h = 22$

$24° + 90° + h° + 2i° = 180°$ (adj. ∠s on a str. line)

$24° + 90° + 22° + 2i° = 180°$

$2i° = 180° - 24° - 90° - 22°$

$= 44°$

$i° = 22°$

$\therefore i = 22$

$j° = 2i°$ (vert. opp. ∠s)

$= 44°$

$\therefore j = 44$

13. (i) $(186 - 4x)° + 34° = 6x°$ (vert. opp. ∠s)

$186° - 4x° + 34° = 6x°$

$6x° + 4x° = 186° + 34°$

$10x° = 220°$

$x° = 22°$

$\therefore x = 22$

$6x° + 3y° = 180°$ (adj. ∠s on a str. line)

$6(22°) + 3y° = 180°$

$132° + 3y° = 180°$

$3y° = 180° - 132°$

$= 48°$

$y° = 16°$

$\therefore y = 16$

(ii) Obtuse $A\hat{O}D = (186 - 4x)° + 34°$

$= [186 - 4(22)]° + 34°$

$= 98° + 34°$

$= 132°$

Reflex $C\hat{O}E = 180° + (186 - 4x)°$

$= 180° + 98°$

$= 278°$

Exercise 10B

1. (a) (i) $B\hat{X}R$ and $D\hat{Z}R$, $A\hat{X}R$ and $C\hat{Z}R$, $A\hat{X}S$ and $C\hat{Z}S$, $B\hat{X}S$ and $D\hat{Z}S$, $B\hat{W}P$ and $D\hat{Y}P$, $A\hat{W}P$ and $C\hat{Y}P$, $A\hat{W}Q$ and $C\hat{Y}Q$, $B\hat{W}Q$ and $D\hat{Y}Q$

(ii) $A\hat{X}S$ and $D\hat{Z}R$, $B\hat{X}S$ and $C\hat{Z}R$, $A\hat{W}Q$ and $D\hat{Y}P$, $B\hat{W}Q$ and $D\hat{Y}P$

(iii) $A\hat{X}S$ and $C\hat{Z}R$, $B\hat{X}S$ and $D\hat{Z}R$, $A\hat{W}Q$ and $C\hat{Y}P$, $B\hat{W}Q$ and $D\hat{Y}P$

(b) No, $B\hat{W}Q \neq A\hat{X}R$ as PQ is not parallel to RS.

(c) No, the sum of $D\hat{Y}P$ and $C\hat{Z}R$ is not equal to 180° as PQ is not parallel to RS.

2. (a) $a° = 117°$ (vert. opp. ∠s)

$\therefore a = 117$

$b° = 117°$ (corr. ∠s, AB // CD)

$\therefore b = 117$

$c° + a° = 180°$ (int. ∠s, AB // CD)

$c° + 117° = 180°$

$c° = 180° - 117°$

$= 63°$

$\therefore c = 63$

$d° = 78°$ (corr. ∠s, AB // CD)

$\therefore d = 78$

(b) $e° = 31°$ (alt. ∠s, AB // CD)

$\therefore e = 31$

$f° = 35° + 31°$ (alt. ∠s, AB // CD)

$= 66°$

$\therefore f = 66$

(c) $g° = 83°$ (alt. ∠s, AB // CD)

$\therefore g = 83$

$h° = 69°$ (corr. ∠s, AB // CD)

$\therefore h = 69$

(d) $i° + 75° + 60° = 180°$ (int. $\angle$s, AB // CD)

$i° = 180° - 75° - 60°$

$= 45°$

$\therefore\ i = 45$

$j° = 60°$ (alt. $\angle$s, AB // CD)

$\therefore\ j = 60$

3. **(a)** $a° = 38°$ (corr. $\angle$s, AB // CD)

$\therefore\ a = 38$

$a° + 30° = 2b°$ (corr. $\angle$s, AB // CD)

$38° + 30° = 2b°$

$2b° = 68°$

$b° = 34°$

$\therefore\ b = 34$

(b) $7c° = 140°$ (corr. $\angle$s, AB // CD)

$c° = 20°$

$\therefore\ c = 20$

$2d° = 7c°$ (vert. opp. $\angle$s)

$= 140°$

$d° = 70°$

$\therefore\ d = 70$

(c) $7e° + 3e° = 180°$ (int. $\angle$s, AB // CD)

$10e° = 180°$

$e° = 18°$

$\therefore\ e = 18$

(d) $(2f + 6)° = (3f - 23)°$ (alt. $\angle$s, AB // CD)

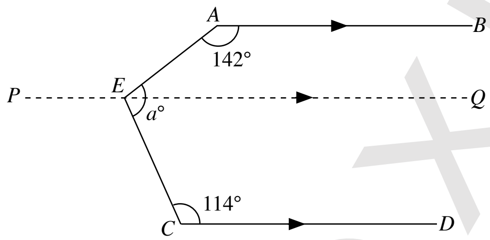
$2f° + 6° = 3f° - 23°$

$3f° - 2f° = 6° + 23°$

$f° = 29°$

$\therefore\ f = 29$

4. **(a)**



$A\hat{E}Q + 142° = 180°$ (int. $\angle$s, AB // PQ)

$A\hat{E}Q = 180° - 142°$

$= 38°$

$C\hat{E}Q + 114° = 180°$ (int. $\angle$s, PQ // CD)

$C\hat{E}Q = 180° - 114°$

$= 66°$

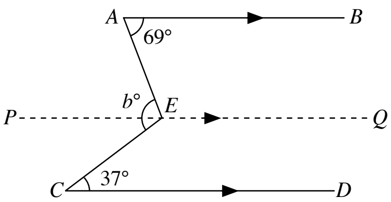
$a° = A\hat{E}Q + C\hat{E}Q$

$= 38° + 66°$

$= 104°$

$\therefore\ a = 104$

(b)



$A\hat{E}P = 69°$ (alt. $\angle$s, AB // PQ)

$C\hat{E}P = 37°$ (alt. $\angle$s, PQ // CD)

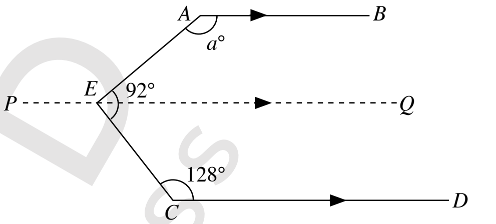
$b° = A\hat{E}P + C\hat{E}P$

$= 69° + 37°$

$= 106°$

$\therefore\ b = 106$

5. **(a)**



$C\hat{E}Q + 128° = 180°$ (int. $\angle$s, PQ // CD)

$C\hat{E}Q = 180° - 128°$

$= 52°$

$A\hat{E}Q = 92° - C\hat{E}Q$

$= 92° - 52°$

$= 40°$

$a° + A\hat{E}Q = 180°$ (int. $\angle$s, AB // PQ)

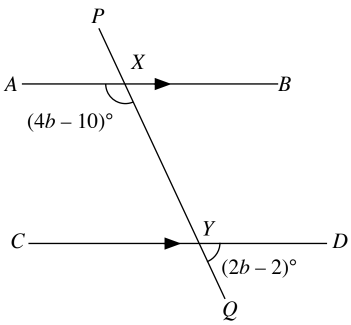
$a° + 40° = 180°$

$a° = 180° - 40°$

$= 140°$

$\therefore\ a = 140$

(b)



$C\hat{Y}P = (2b - 2)°$ (vert. opp. $\angle$s)

$C\hat{Y}P + (4b - 10)° = 180°$ (int. $\angle$s, AB // CD)

$(2b - 2)° + (4b - 10)° = 180°$

$2b° - 2° + 4b° - 10° = 180°$

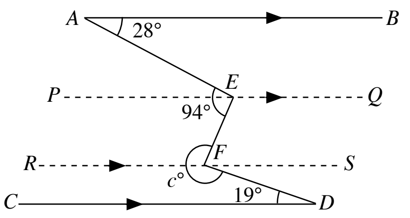
$2b° + 4b° = 180° + 2° + 10°$

$6b° = 192°$

$b° = 32$

$\therefore\ b = 32$

(c)



$A\hat{E}P = 28^\circ$ (alt. $\angle$s, $AB \,//\, PQ$)

$F\hat{E}P = 94^\circ - A\hat{E}P$

$= 94^\circ - 28^\circ$

$= 66^\circ$

$E\hat{F}S = F\hat{E}P$

$= 66^\circ$ (alt. $\angle$s, $PQ \,//\, RS$)

$D\hat{F}S = 19^\circ$ (alt. $\angle$s, $RS \,//\, CD$)

$c^\circ + E\hat{F}S + D\hat{F}S = 360^\circ$ ($\angle$s at a point)

$c^\circ + 66^\circ + 19^\circ = 360^\circ$

$c^\circ = 360^\circ - 66^\circ - 19^\circ$

$= 275^\circ$

$\therefore c = 275$

6. **(i)** $C\hat{D}F = 86^\circ$ (alt. $\angle$s, $CE \,//\, FG$)

(ii) $H\hat{D}E = 86^\circ$ (vert. opp. $\angle$s)

$E\hat{D}A = H\hat{D}E - 47^\circ$

$= 86^\circ - 47^\circ$

$= 39^\circ$

$B\hat{A}D + E\hat{D}A = 180^\circ$ (int. $\angle$s, $AB \,//\, CE$)

$B\hat{A}D + 39^\circ = 180^\circ$

$B\hat{A}D = 180^\circ - 39^\circ$

$= 141^\circ$

7. **(i)** $A\hat{E}B = 68^\circ$ (alt. $\angle$s, $BF \,//\, AD$)

(ii) $E\hat{A}B = 58^\circ$ (alt. $\angle$s, $AB \,//\, CD$)

$F\hat{B}A + E\hat{A}B = 180^\circ$ (int. $\angle$s, $BF \,//\, AD$)

$F\hat{B}A + 58^\circ = 180^\circ$

$F\hat{B}A = 180^\circ - 58^\circ$

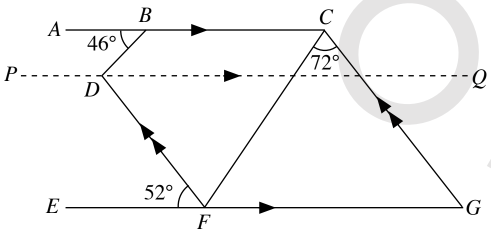
$= 122^\circ$

$A\hat{B}E = F\hat{B}A - 68^\circ$

$= 122^\circ - 68^\circ$

$= 54^\circ$

8.



(i) $C\hat{G}F = 52^\circ$ (corr. $\angle$s, $FD \,//\, GC$)

(ii) $B\hat{C}G + C\hat{G}F = 180^\circ$ (int. $\angle$s, $AC \,//\, EG$)

$B\hat{C}G + 52^\circ = 180^\circ$

$B\hat{C}G = 180^\circ - 52^\circ$

$= 128^\circ$

$B\hat{C}F = B\hat{C}G - 72^\circ$

$= 128^\circ - 72^\circ$

$= 56^\circ$

(iii) $B\hat{D}Q = 46^\circ$ (alt. $\angle$s, $AC \,//\, PQ$)

$F\hat{D}Q = 52^\circ$ (alt. $\angle$s, $PQ \,//\, EG$)

Reflex $B\hat{D}F + B\hat{D}Q + F\hat{D}Q = 360^\circ$ ($\angle$s at a point)

Reflex $B\hat{D}F + 46^\circ + 52^\circ = 360^\circ$

Reflex $B\hat{D}F = 360^\circ - 46^\circ - 52^\circ$

$= 262^\circ$

9. $F\hat{D}C + 58^\circ = 180^\circ$ (adj. $\angle$s on a str. line)

$F\hat{D}C = 180^\circ - 58^\circ$

$= 122^\circ$

$D\hat{C}A = F\hat{D}C$ (alt. $\angle$s, $DF \,//\, AC$)

$= 122^\circ$

$D\hat{C}A + 4x^\circ = 360^\circ$ ($\angle$s at a point)

$122^\circ + 4x^\circ = 360^\circ$

$4x^\circ = 360^\circ - 122^\circ$

$= 238^\circ$

$x^\circ = 59.5^\circ$

$\therefore x = 59.5$

$B\hat{A}C + D\hat{C}A = 180^\circ$ (int. $\angle$s, $AB \,//\, CE$)

$B\hat{A}C + 122^\circ = 180^\circ$

$B\hat{A}C = 180^\circ - 122^\circ$

$= 58^\circ$

$7y^\circ + B\hat{A}C = 360^\circ$

$7y^\circ + 58^\circ = 360^\circ$

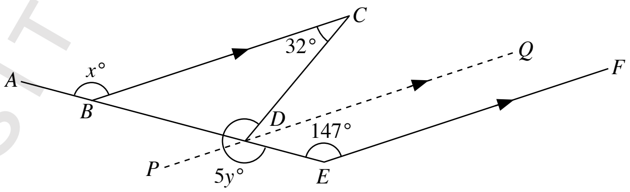
$7y^\circ = 360^\circ - 58^\circ$

$= 302^\circ$

$y^\circ = 43.1^\circ$ (to 1 d.p.)

$\therefore y = 43.1$

10.



$x^\circ = 147^\circ$ (corr. $\angle$s, $BC \,//\, EF$)

$\therefore x = 147$

$C\hat{D}Q = 32^\circ$ (alt. $\angle$s, $BC \,//\, PQ$)

$Q\hat{D}E + 147^\circ = 180^\circ$ (int. $\angle$s, $PQ \,//\, EF$)

$Q\hat{D}E = 180^\circ - 147^\circ$

$= 33^\circ$

$5y^\circ + C\hat{D}Q + Q\hat{D}E = 360^\circ$ ($\angle$s at a point)

$5y^\circ + 32^\circ + 33^\circ = 360^\circ$

$5y^\circ = 360^\circ - 32^\circ - 33^\circ$

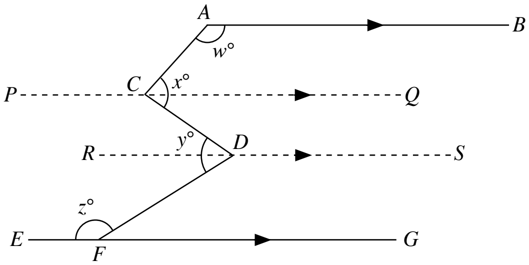
$= 295^\circ$

$y^\circ = 59^\circ$

$\therefore y = 59$

11. Since $A\hat{X}S + C\hat{Z}R = 104^\circ + 76^\circ = 180^\circ$,
then $AB \,//\, CD$ (converse of int. $\angle$s).
$\therefore B\hat{W}P = D\hat{Y}P = 46^\circ$ (corr. $\angle$s, $AB \,//\, CD$)

12.



$Q\hat{C}A + w° = 180°$ (int. $\angle$s, AB // PQ)

$Q\hat{C}A = 180° - w°$

$Q\hat{C}D = x° - Q\hat{C}A$

$= x° - (180° - w°)$

$= x° - 180° + w°$

$C\hat{D}R = Q\hat{C}D$ (alt. $\angle$s, PQ // RS)

$= x° - 180° + w°$

$F\hat{D}R = y° - C\hat{D}R$

$= y° - (x° - 180° + w°)$

$= y° - x° + 180° - w°$

$F\hat{D}R + z° = 180°$ (int. $\angle$s, RS // EG)

$y° - x° + 180° - w° + z° = 180°$

$w° + x° = y° + z°$

$\therefore\ w + x = y + z$

Review Exercise 10

1. **(a)** $32° + 4a° + 84° = 180°$ (adj. $\angle$s on a str. line)

$4a° = 180° - 32° - 84°$

$= 64°$

$a° = 16°$

$\therefore\ a = 16$

$84° + 2b° = 180°$ (adj. $\angle$s on a str. line)

$2b° = 180° - 84°$

$= 96°$

$b° = 48°$

$\therefore\ b = 48$

(b) $c° + 68° = 180°$ (adj. $\angle$s on a str. line)

$c° = 180° - 68°$

$= 112°$

$\therefore\ c = 112$

$68° + 3d° - 5° + 30° = 180°$ (adj. $\angle$s on a str. line)

$3d° = 180° - 68° + 5° - 30°$

$= 87°$

$d° = 29°$

$\therefore\ d = 29$

2. **(a)** $4a° + 2a° + a° + a° + 2a° = 360°$ ($\angle$s at a point)

$10a° = 360°$

$a° = 36°$

$\therefore\ a = 36$

(b) $(3b - 14)° + (4b - 21)° + (2b + 1)° + (b + 34)°$

$= 360°$ ($\angle$s at a point)

$3b° - 14° + 4b° - 21° + 2b° + 1° + b° + 34° = 360°$

$3b° + 4b° + 2b° + b° = 360° + 14° + 21° - 1° - 34°$

$10b° = 360°$

$b° = 36°$

$\therefore\ b = 36$

3. **(a)** $C\hat{O}F = 4a° - 17°$ (vert. opp. $\angle$s)

$2a° + C\hat{O}F + 3a° - 10° = 180°$ (adj. $\angle$s on a str. line)

$2a° + 4a° - 17° + 3a° - 10° = 180°$

$2a° + 4a° + 3a° = 180° + 17° + 10°$

$9a° = 207°$

$a° = 23°$

$\therefore\ a = 23$

(b) $C\hat{O}F = 2b° + 15°$ (vert. opp. $\angle$s)

$2b° + 2b° + 15° + b° = 180°$ (adj. $\angle$s on a str. line)

$2b° + 2b° + b° = 180° - 15°$

$5b° = 165°$

$b° = 33°$

$\therefore\ b = 33$

$b° + 3c° + 2b° + 15° = 180°$ (adj. $\angle$s on a str. line)

$33° + 3c° + 2(33°) + 15° = 180°$

$33 + 3c° + 66° + 15° = 180°$

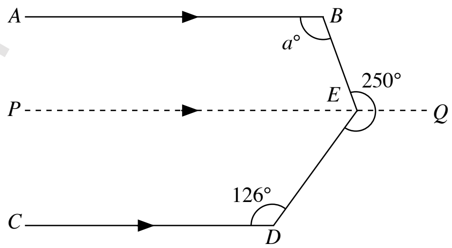
$3c° = 180° - 33° - 66° - 15°$

$= 66°$

$c° = 22°$

$\therefore\ c = 22$

4. **(a)**



$D\hat{E}P + 126° = 180°$ (int. $\angle$s, PQ // CD)

$D\hat{E}P = 180° - 126°$

$= 54°$

$B\hat{E}P + D\hat{E}P + 250° = 360°$ ($\angle$s at a point)

$B\hat{E}P + 54° + 250° = 360°$

$B\hat{E}P = 360° - 54° - 250°$

$= 56°$

$a° + B\hat{E}P = 180°$ (int. $\angle$s, AB // PQ)

$a° + 56° = 180°$

$a° = 180° - 56°$

$= 124°$

$\therefore\ a = 124$

(b) $(6b - 21)° + (5b - 52)° = 180°$ (int. ∠s, $AB // CD$)

$6b° - 21° + 5b° - 52° = 180°$

$6b° + 5b° = 180° + 21° + 52°$

$11b° = 253°$

$b° = 23°$

$\therefore b = 23$

$3c° = (6b - 21)°$

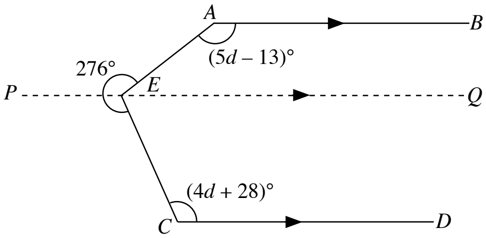
$= [6(23) - 21]°$

$= 117°$

$c° = 39°$

$\therefore c = 39$

(c)



$Q\hat{E}A + (5d - 13)° = 180°$ (int. ∠s, $AB // PQ$)

$Q\hat{E}A + 5d° - 13° = 180°$

$Q\hat{E}A = 180° - 5d° + 13°$

$= 193° - 5d°$

$Q\hat{E}C + (4d + 28)° = 180°$ (int. ∠s, $PQ // CD$)

$Q\hat{E}C + 4d° + 28° = 180°$

$Q\hat{E}C = 180° - 4d° - 28°$

$= 152° - 4d°$

$276° + Q\hat{E}A + Q\hat{E}C = 360°$ (∠s at a point)

$276° + 193° - 5d° + 152° - 4d° = 360°$

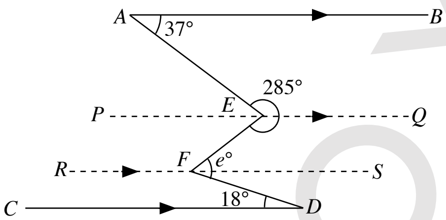
$5d° + 4d° = 276° + 193° + 152° - 360°$

$9d° = 261°$

$d° = 29°$

$\therefore d = 29$

(d)



$A\hat{E}P = 37°$ (alt. ∠s, $AB // PQ$)

$A\hat{E}P + 285° + F\hat{E}P = 360°$ (∠s at a point)

$37° + 285° + F\hat{E}P = 360°$

$F\hat{E}P = 360° - 37° - 285°$

$= 38°$

$E\hat{F}S = 38°$ (alt. ∠s, $PQ // RS$)

$D\hat{F}S = 18°$ (alt. ∠s, $RS // CD$)

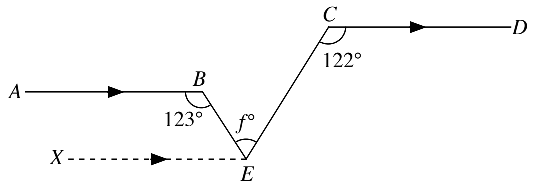
$e° = E\hat{F}S + D\hat{F}S$

$= 38° + 18°$

$= 56°$

$\therefore e = 56$

(e)



$X\hat{E}C = 122°$ (alt. ∠s, $CD // XE$)

$X\hat{E}B + 123° = 180°$ (int. ∠s, $AB // XE$)

$X\hat{E}B = 180° - 123°$

$= 57°$

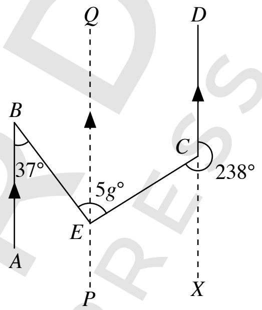
$f° = X\hat{E}C - X\hat{E}B$

$= 122° - 57°$

$= 65°$

$\therefore f = 65$

(f)



$B\hat{E}Q = 37°$ (alt. ∠s, $AB // PQ$)

$E\hat{C}X = 238° - 180°$

$= 58°$

$C\hat{E}Q = E\hat{C}X$ (alt. ∠s, $PQ // XD$)

$= 58°$

$5g° = B\hat{E}Q + C\hat{E}Q$

$= 37° + 58°$

$= 95°$

$g° = 19°$

$\therefore g = 19$

5. **(i)** $C\hat{D}F = 148°$ (alt. ∠s, $GC // DF$)

$C\hat{D}E + 84° + C\hat{D}F = 360°$ (∠s at a point)

$C\hat{D}E + 84° + 148° = 360°$

$C\hat{D}E = 360° - 84° - 148°$

$= 128°$

(ii) $A\hat{B}C = C\hat{D}E$ (alt. ∠s, $AB // DE$)

$= 128°$

$A\hat{B}H = A\hat{B}C - 74°$

$= 128° - 74°$

$= 54°$

6. **(i)** $D\hat{E}H + 26° = 180°$ (adj. ∠s on a str. line)

$D\hat{E}H = 180° - 26°$

$= 154°$

(ii) $B\hat{E}H = 62°$ (alt. ∠s, $EC // GH$)

$D\hat{E}B + B\hat{E}H + D\hat{E}H = 360°$ (∠s at a point)

$D\hat{E}B + 62° + 154° = 360°$

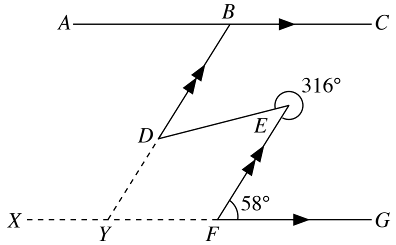
$D\hat{E}B = 360° - 62° - 154°$

$= 144°$

$A\hat{B}C = D\hat{E}B$ (corr. ∠s, $AB // DF$)

$= 144°$

7.



(i) $D\hat{E}F$ + reflex $D\hat{E}F = 360°$ (∠s at a point)

$D\hat{E}F + 316° = 360°$

$D\hat{E}F = 360° - 316°$

$= 44°$

$B\hat{D}E = D\hat{E}F$ (alt. ∠s, DB // FE)

$= 44°$

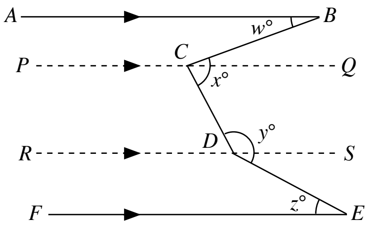
(ii) $D\hat{Y}F = 58°$ (corr. ∠s, YB // FE)

$A\hat{B}D = D\hat{Y}F$ (alt. ∠s, AC // XG)

$= 58°$

8. Since $B\hat{W}Q + D\hat{Y}P = 123° + 57° = 180°$,
then AB // CD (converse of int. ∠s).
∴ $D\hat{Z}R = A\hat{X}S = 118°$ (alt. ∠s, AB // CD)

9.



$B\hat{C}Q = w°$ (alt. ∠s, AB // PQ)

$D\hat{C}Q = x° - B\hat{C}Q$

$= x° - w°$

$C\hat{D}S + D\hat{C}Q = 180°$ (int. ∠s, PQ // RS)

$C\hat{D}S + x° - w° = 180°$

$C\hat{D}S = 180° - x° + w°$

$E\hat{D}S = y° - (180° - x° + w°)$

$= y° - 180° + x° - w°$

$E\hat{D}S = z°$ (alt. ∠s, RS // FE)

$y° - 180° + x° - w° = z°$

$w° + z° + 180° = x° + y°$

∴ $w + z + 180 = x + y$

Challenge Yourself

1.

Number of Rays between OA and OB	Number of Different Angles
0	$1 = \frac{1}{2} \times 1 \times 2$
1	$3 = \frac{1}{2} \times 2 \times 3$
2	$6 = \frac{1}{2} \times 3 \times 4$
3	$10 = \frac{1}{2} \times 4 \times 5$
4	$15 = \frac{1}{2} \times 5 \times 6$
⋮	⋮
n	$\frac{1}{2}(n+1)(n+2)$

Number of different angles in the figure $= \frac{1}{2}(n+1)(n+2)$

2. Angle hour hand moves in 1 hour $= \frac{1}{12} \times 360°$

$= 30°$

Angle hour hand moves from 12 noon to 7 p.m. $= 7 \times 30°$

$= 210°$

Angle hour hand moves from 7 p.m. to 7.20 p.m. $= \frac{20}{60} \times 30°$

$= 10°$

Angle hour hand moves from 12 noon to 7.20 p.m. $= 210° + 10°$

$= 220°$

Angle minute hand moves from 7 p.m. to 7.20 p.m. $= \frac{20}{60} \times 360°$

$= 120°$

Smaller angle between minute hand and hour hand at 7.20 p.m.
$= 220° - 120°$
$= 100°$

Larger angle between minute hand and hour hand at 7.20 p.m.
$= 360° - 100°$ (∠s at a point)
$= 260°$

3. From 9 a.m. to before 9 p.m. on any particular day, the bell will sound twice every hour, except for the hour from 1 p.m. to before 2 p.m. and the hour from 2 p.m. to before 3 p.m., when it only sounds once during each hour.

Likewise, from 9 p.m. on any particular day to before 9 a.m. the next day, the bell will sound twice every hour, except for the hour from 1 a.m. to before 2 a.m. and the hour from 2 a.m. to before 3 a.m, when it only sounds once during each hour.

∴ Number of times bell will sound from 9 a.m. on a particular day to before 9 p.m. the next day
$= 2 \times 30 + 1 \times 6$
$= 60 + 6$
$= 66$

Since the bell will sound at 9 p.m. the next day,
Number of times bell will sound from 9 a.m. on a particular day to 9 p.m. the next day
$= 66 + 1$
$= 67$

Chapter 11 Geometrical Constructions

TEACHING NOTES

Suggested Approach

Students have learnt how to draw triangles and quadrilaterals using rulers, protractors and set squares in primary school. Teachers need to reintroduce these construction tools and demonstrate the use of these if students are still unfamiliar with them. When students are comfortable with the use of these construction tools and the compasses, teachers can proceed to the sections on construction of triangles and quadrilaterals.

Section 11.1: Introduction to Geometrical Constructions

Teachers may wish to recap with students how rulers, protractors and set squares are used. More emphasis should be placed on the use of protractors, such as the type of scale (inner or outer) to use, depending on the type of angle (acute or obtuse). Teachers need to impress upon students to avoid parallax errors when reading the length using a ruler, or an angle using a protractor.

Teachers should show and lead students on the use of compasses. Students are to know and be familiar with the useful tips in using the construction tools.

Section 11.2: Perpendicular Bisectors and Angle Bisectors

Teachers should state and define perpendicular bisectors and angle bisectors. Stating what perpendicular and bisect means individually will help students to remember their meanings.

For the worked examples in this section, teachers are encouraged to go through the construction steps one by one with the students. Students should follow and construct the same figures as shown in the worked examples.

Teachers should allow students to use suitable geometry software to explore and discover the properties of perpendicular bisectors and angle bisectors (see Investigation: Property of a Perpendicular Bisector and Investigation: Property of an Angle Bisector), that is, their equidistance from end-points and sides of angles respectively.

WORKED SOLUTIONS

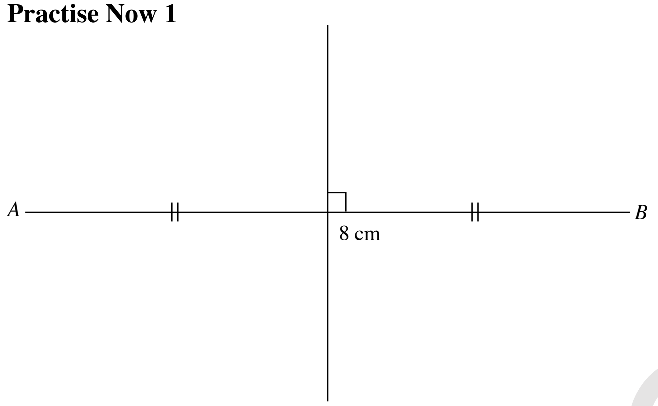
Investigation (Property of a Perpendicular Bisector)

4. The length of *AC* is equal to the length of *BC*.
5. Any point on the perpendicular bisector of *AB* is equidistant from *A* and *B*.
6. Any point which is not on the perpendicular bisector of *AB* is not equidistant from *A* and *B*.

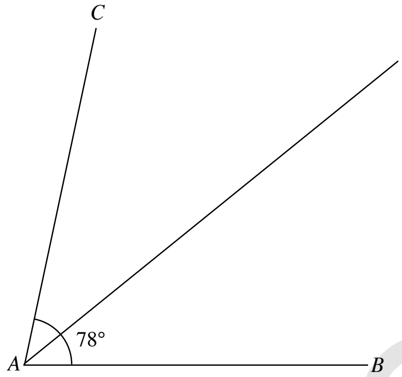
Investigation (Property of an Angle Bisector)

5. The length of *PR* is equal to the length of *QR*.
6. Any point on the angle bisector of $B\hat{A}C$ is equidistant from *AB* and *AC*.
7. Any point which is not on the angle bisector of $B\hat{A}C$ is not equidistant from *AB* and *AC*.

Practise Now 1

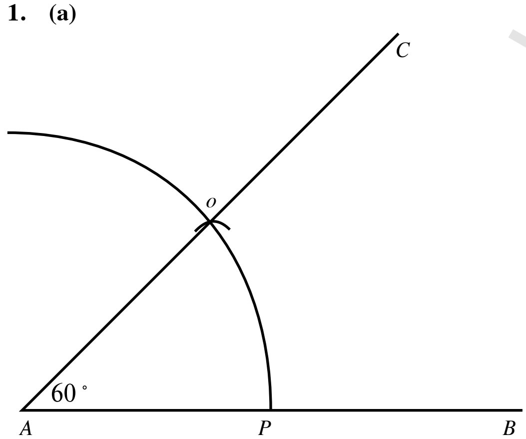


Practise Now 2



Practise Now 3

1. **(a)**



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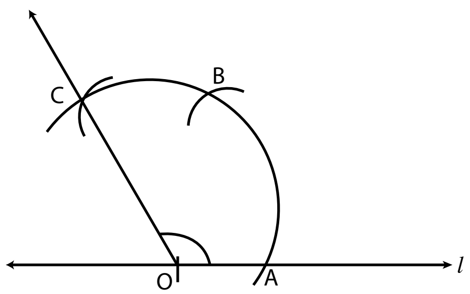
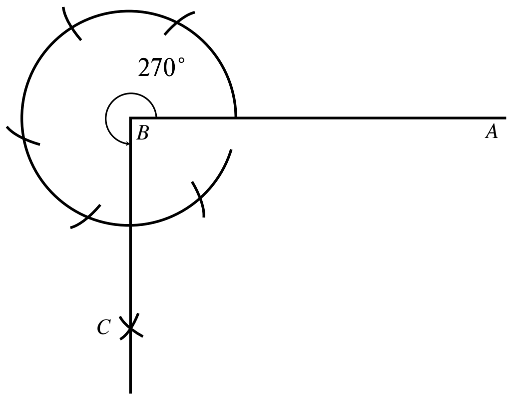
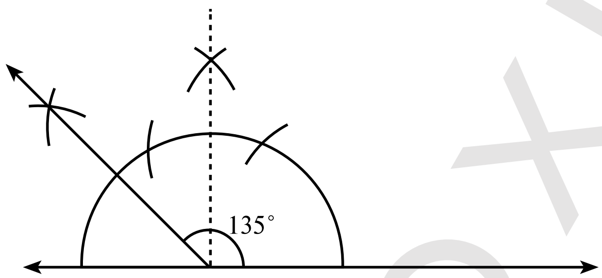


(c)



Practise Now 4

1. **(a)**



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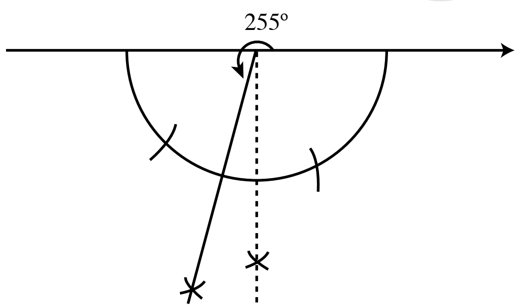

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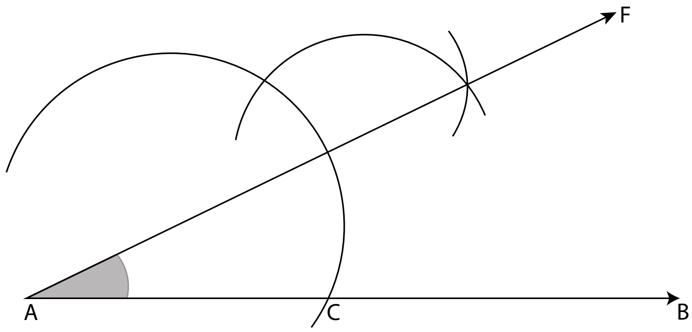
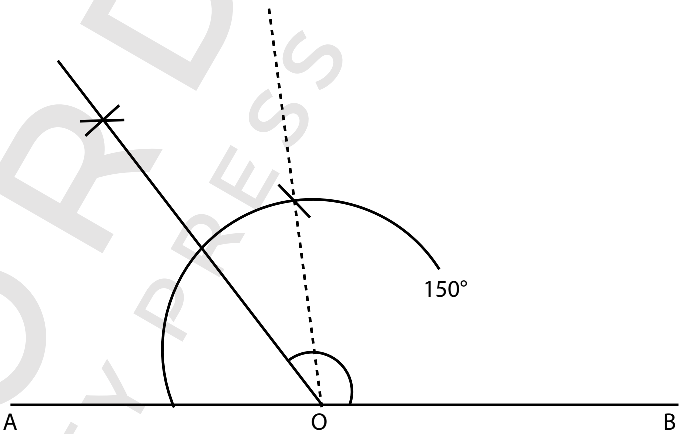
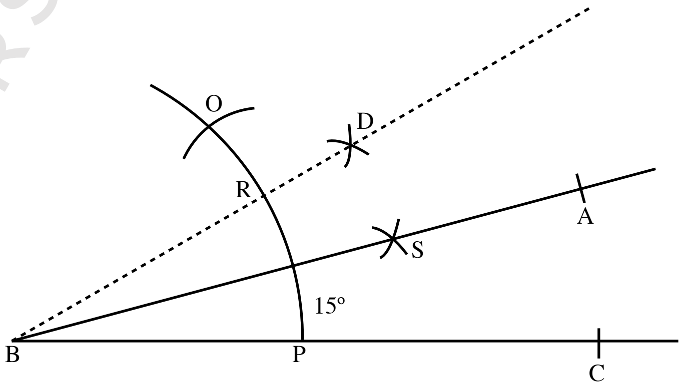


Practise Now 5

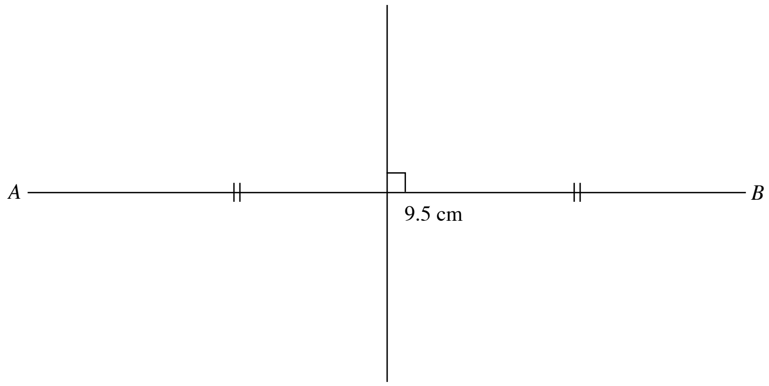


Practise Now 6

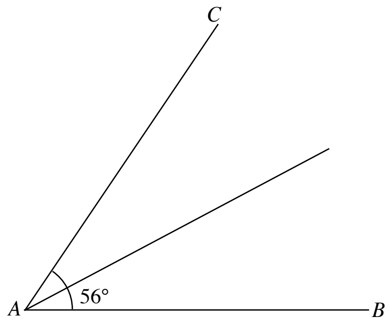


Exercise 11

1.

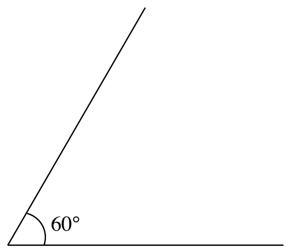


2.

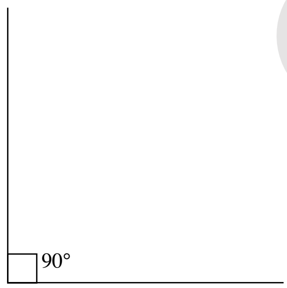


3.

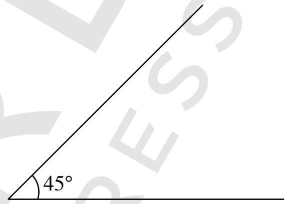
(i)



(ii)



(iii)



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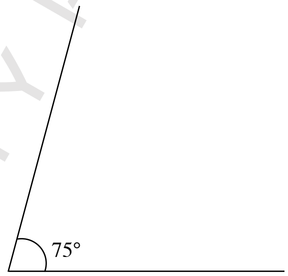

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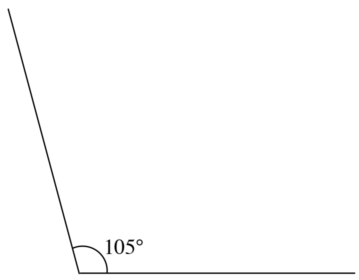

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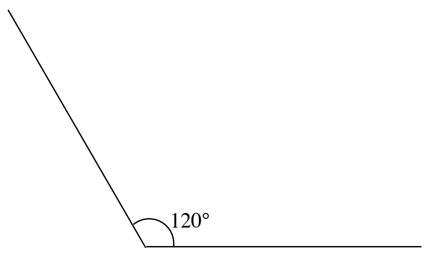
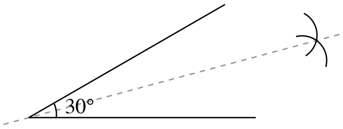
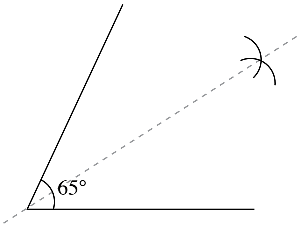


4.

(i)



(ii)



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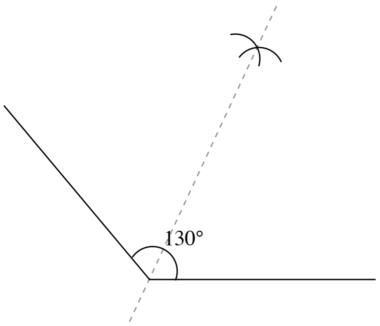

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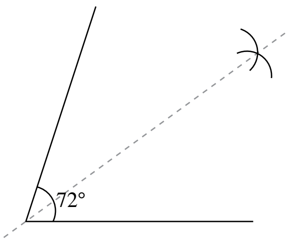

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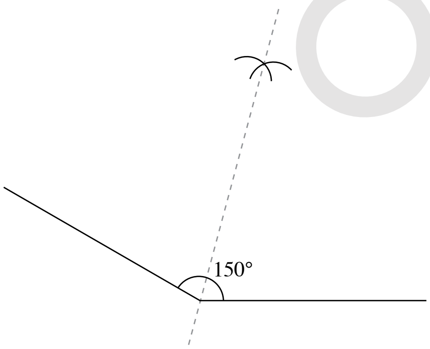
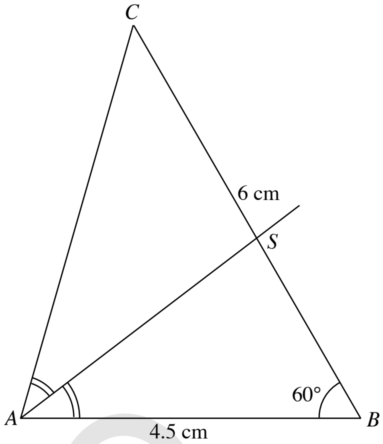


Review Exercise 11

1.



(i) Length of AC = 5.4 cm

(ii) Length of CS = 3.3 cm

2. (i) Steps of construction:

Step 1: Draw a line segment $\overline{AB}$ = 5 cm

Step 2: With A as centre and a radius more than half of $\overline{AB}$, draw two arcs, one on each side of AB as shown.

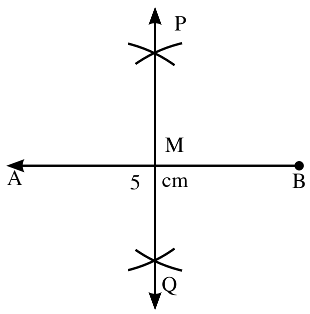
Step 3: With B as centre and with the same radius as before, draw two more arcs to cut the previous arcs at P and Q.

Step 4: Join P to Q. Produce $\overline{PQ}$ in both directions to form $\overleftrightarrow{PQ}$.

Step 5: Measure $\overline{AM}$ and $\overline{MB}$.

$\overline{AM} = \overline{MB}$ = 2.5 cm

Therefore, $\overleftrightarrow{PQ}$ bisects the line segment $\overline{AB}$ at M.



Follow the same steps in

(ii), (iii), (iv) and (v)

3. (i) Steps of construction:

Step 1: Draw $\overline{AB}$ = 4 cm. Mark a point N on it.

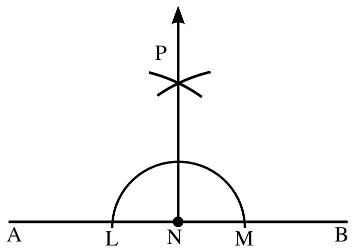
Step 2: With N as centre and with a suitable radius, draw an arc to intersect $\overline{AB}$ at L and M.

Step 3: With L as centre and a radius of more than $\overline{LM}$, draw an arc above AB.

Step 4: With M as centre and the same radius, draw another arc to intersect the previous arc at P.

Step 5: Join P and N.

$\overline{NP}$ is the required perpendicular to $\overline{AB}$



4 cm

Follow the same steps of construction for:

(ii), (iii), (iv) and (v)

4. (i) Steps of construction:

Steps 1: Draw a line segment

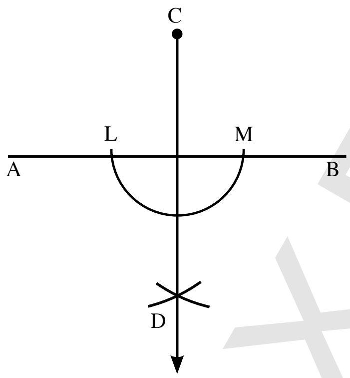
$\overline{AB}$ = 9 cm. Take a point C lying outside and above it.

Step 2: With C as centre and with a suitable radius, draw an arc to intersect $\overline{AB}$ at L and M.

Step 3: With L as centre, draw an arc with radius greater than half of $\overline{LM}$.

Step 4: With M as centre and the same radius, draw another arc to intersect the previous arc at D.

Step 5: Draw a line through C and D.



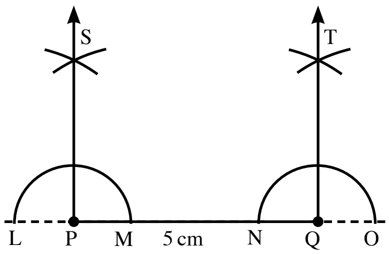
$\overline{CD}$ is the required perpendiuclar to $\overline{AB}$.

5. Steps of construction:

Step 1: Draw a line segment

$\overline{PQ}$ = 5 cm.

Step 2: Follow the steps of construction of drawing a perpendicular as given in Q4 (i).



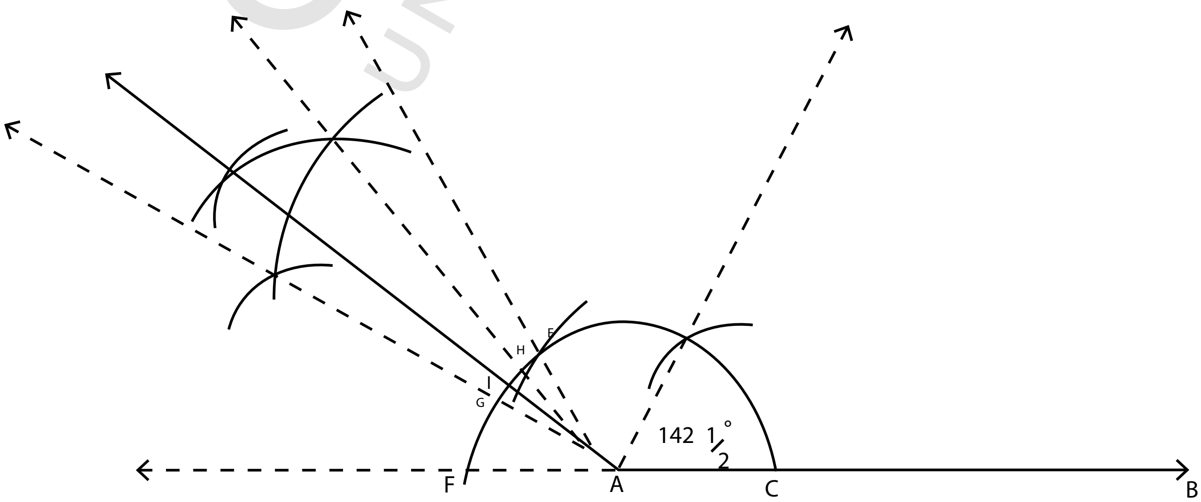
$\overline{PS}$ is a perpendicular to $\overline{PQ}$ at point P.

Simlarly, $\overline{QT}$ is a perpendicular to $\overline{PQ}$ at point Q. The distance between the two perpendiculars $\overline{PQ}$ and $\overline{QT}$ is same, that is 5 cm.

Therefore, $\overline{PQ}$ is parallel to $\overline{QT}$.

Challenge Yourself

1.



Chapter 12 Symmetry

TEACHING NOTES

Suggested Approach:

Many buildings and objects in our surroundings are symmetrical in shape. Teachers can make use of these real life examples to allow students to appreciate the significance of symmetry in the way things are designed (see Chapter Opener on Page 355). Teachers can also highlight that this is not the first contact students have with symmetry, as most animals and insects, and even our faces, are symmetrical in shape. Students can be encouraged to think about symmetry around them and if there are any animals that are asymmetrical (see Thinking Time on Page 203).

Section 12.1: Line Symmetry

Teachers can point out objects in the classroom with symmetrical designs, such as windows, desks and chairs. Teachers can also ask students to think about other symmetrical objects they come across daily, such as buildings or company logos etc. The link to reflection, which students have learnt in Chapter 11, can be emphasised, whereby symmetry implies that folding along the line of symmetry will give the same exact shape (see Investigation on page 202).

Section 12.2: Rotational Symmetry in Plane Figures

Teachers may bring in plane rectangles, parallelograms or squares to illustrate to students the concept of rotational symmetry. Familiar objects such as the King, Queen and Jack picture cards can be used to enhance association and appreciation of rotational symmetry of order 2.

WORKED SOLUTIONS

Investigation (Line Symmetry in Two Dimensions)

5. The reflected image in the mirror is the same "half-shape" as the original figure on the paper.

Thinking Time (Page 203)

Most human beings and animals are symmetrical.
An example of an asymmetrical animal is the sole fish, which is edible and can be found in markets. The upper side of its body is a dark greyish brown with thicker flesh, while the underside is white with thinner flesh. Some other animals that appear asymmetrical include hermit crabs which have claws of different sizes, flatfish such as flounders which have their eyes on only one side of their bodies and the wrybill, a bird with a beak bent towards the right.

Thinking Time (Page 204)

No. Triangles that have lines of symmetry are equilateral triangles (3 lines) and isosceles triangles (1 line).

Investigation (Rotational Symmetry in Two Dimensions)

2. $\frac{1}{3}$ of a complete turn

3. $\frac{2}{3}$ of a complete turn from the original position

4. 3 thirds will have to be made for the figure to be back in its original position.

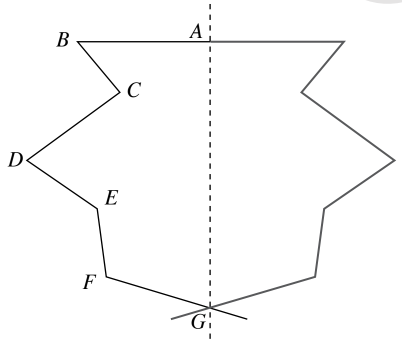
Class Discussion (Line and Rotational Symmetry in Circles)

1. Teachers can highlight to students to observe that a circle can be folded in half in an infinite number of ways, and this means that it has infinite lines of symmetry.

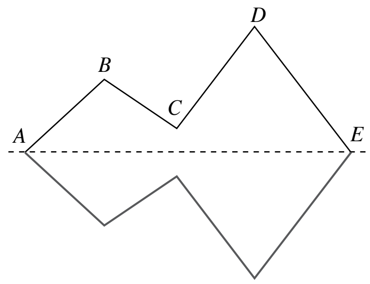
2. To determine rotational symmetry, teachers can prompt students to use a pen to hold the circle down in the centre while rotating it on the table. The shape of the circle will always remain, and hence the order of rotational symmetry is infinite.

Practise Now 1

(a)

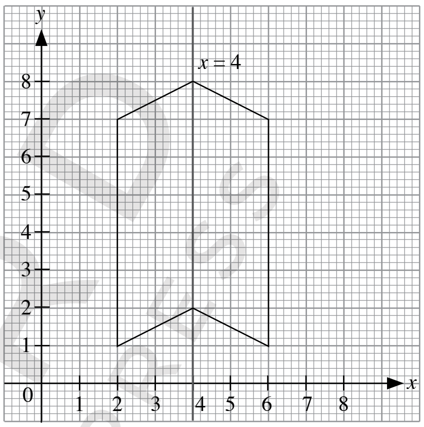


(b)



Practise Now 2

(a)



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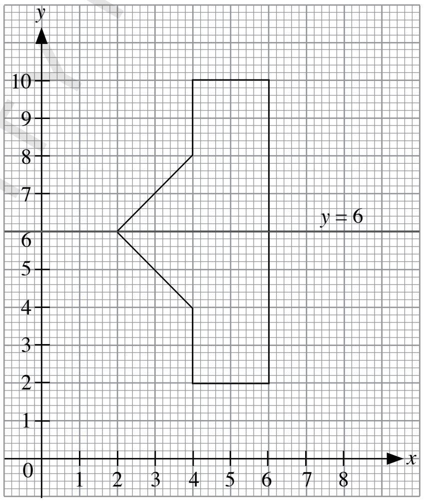
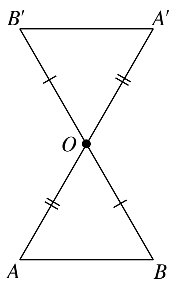
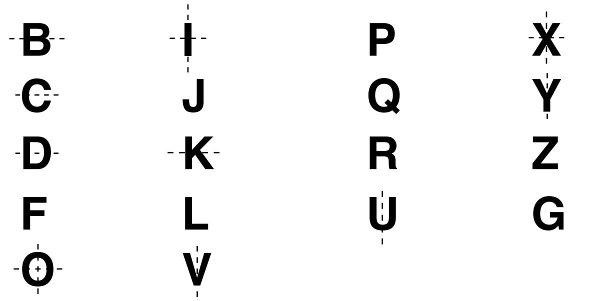
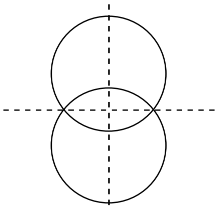


Practise Now 3



Exercise 12A

1. (a), (c), (g)
2. **(a)** 1
   (b) 1
   (c) 1
   (d) 3
   (e) 2
   (f) 2
   (g) 2
   (h) 2
   (i) 4
   (j) 2
   (k) 1
3. 

4. **(a)**

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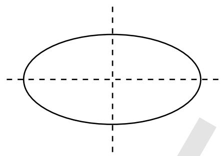
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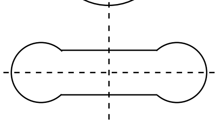
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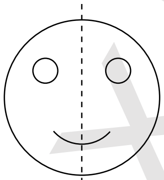
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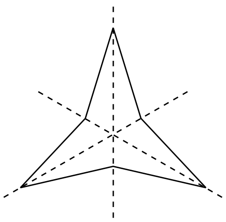
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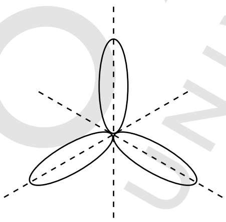
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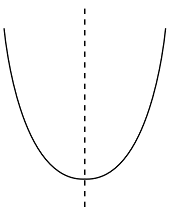
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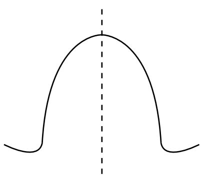
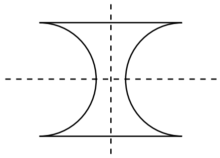

(i)

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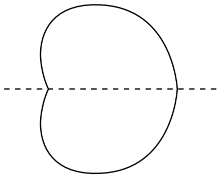
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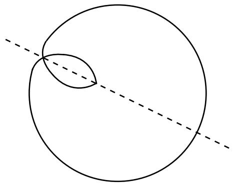
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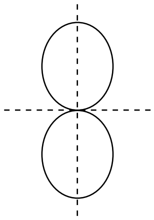
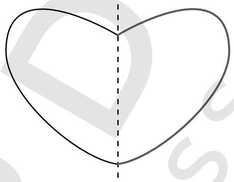
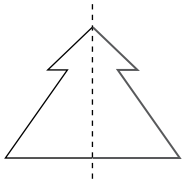


5. **(a)**

(b)

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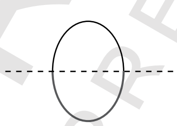
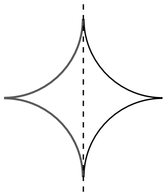

(d)

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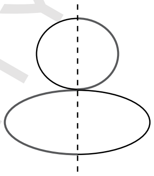
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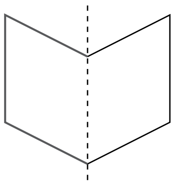
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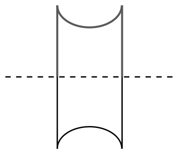
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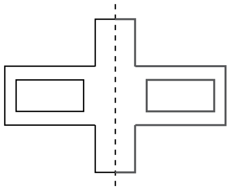
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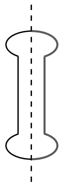
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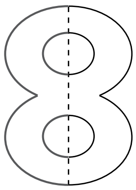
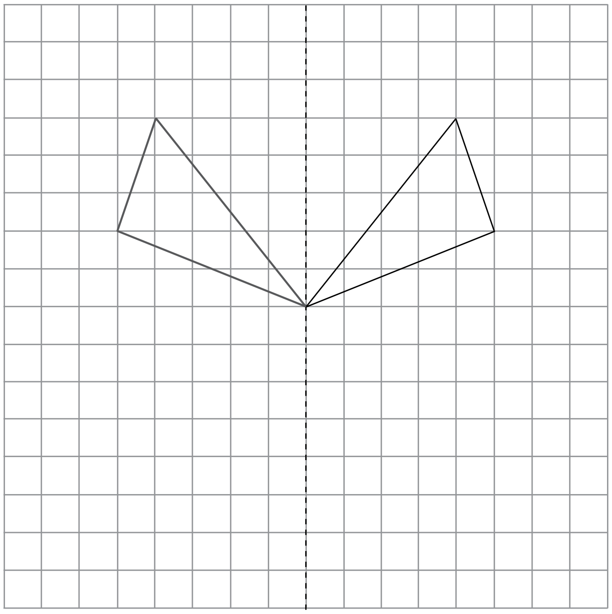


6. (a)



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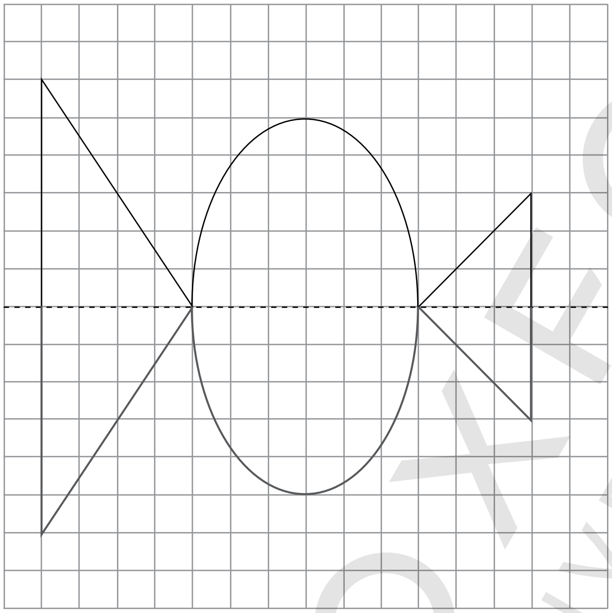

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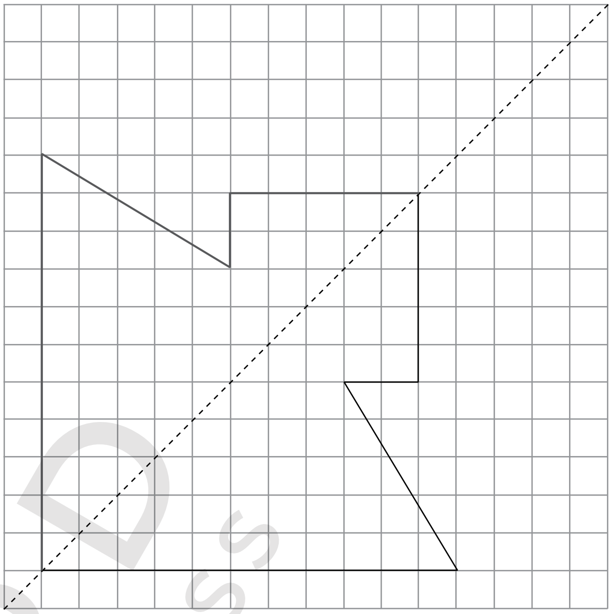
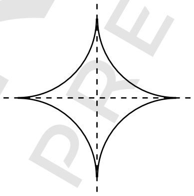


7. (a)



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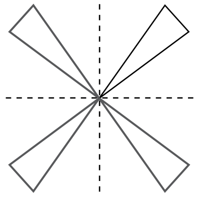

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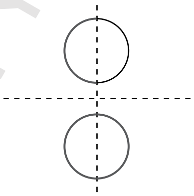

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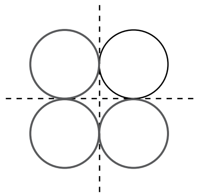

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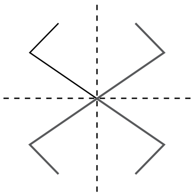

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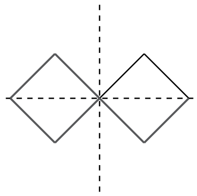

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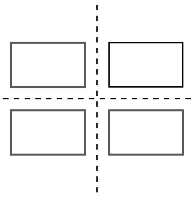
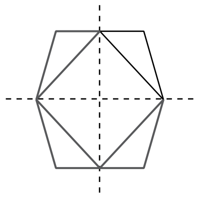
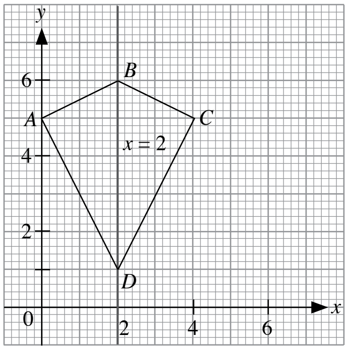


(h)



8. **(a)**



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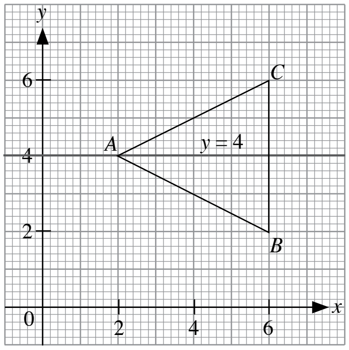

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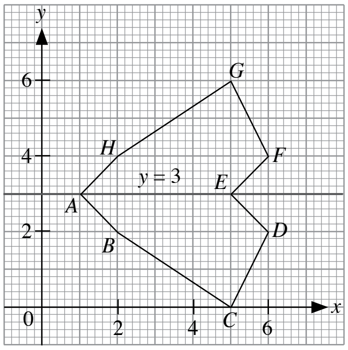

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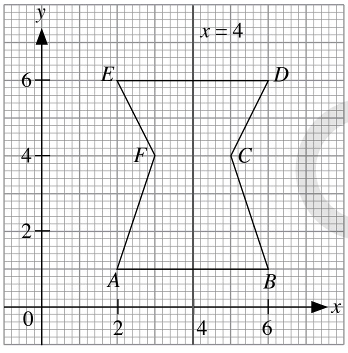
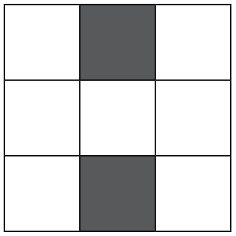
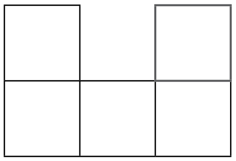


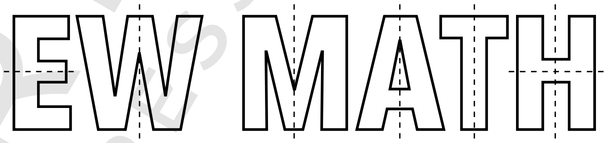
9.



10.

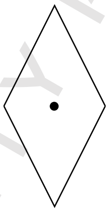


11. **(a)** W, M, A, T, H
(b) E, H
(c) H
(d) N, S
(e)

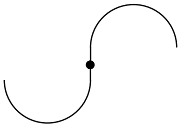


Exercise 12B

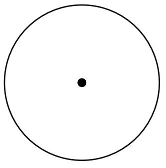
1. **(a)**



(b)



(c)



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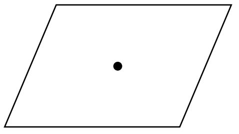
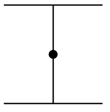


(e)



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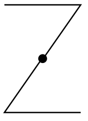

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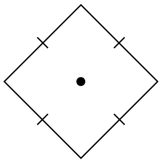

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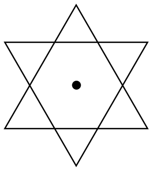
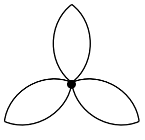
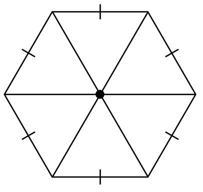


(i)



(j)

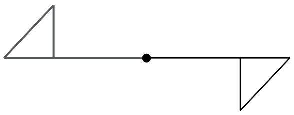


2. Since the figure can be rotated 5 times to fit the original figure, $r = 5$.

3.

	(i) Lines of symmetry	(ii) Order of rotational symmetry
(a)	1	1
(b)	2	2
(c)	0	2
(d)	8	8
(e)	2	2
(f)	2	2
(g)	2	2
(h)	2	2
(i)	0	4

4. (a)



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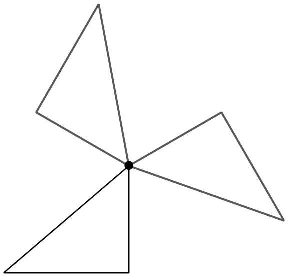

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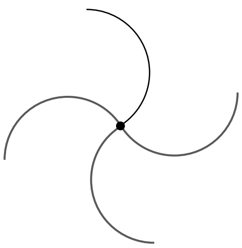
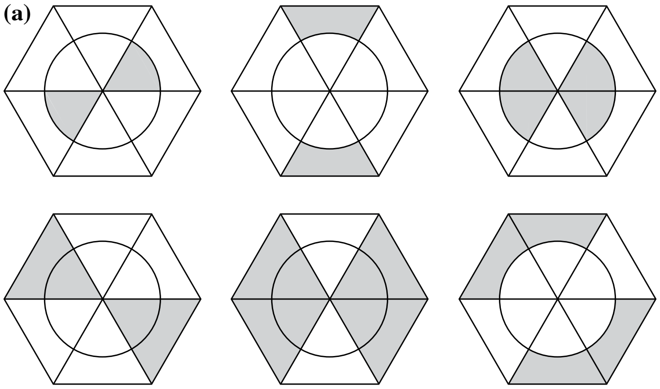
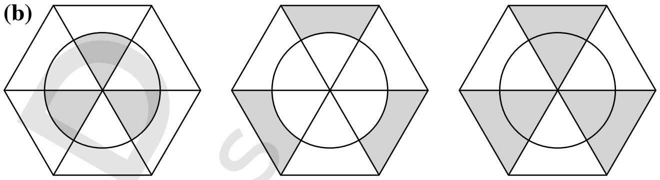


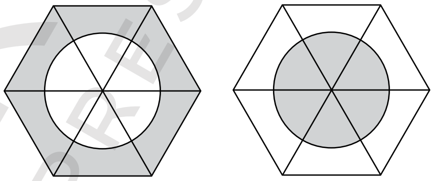
5. (a)



(b)

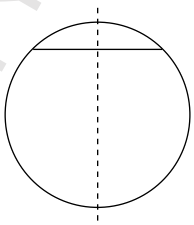


(c)

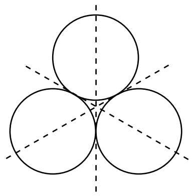


Review Exercise 12

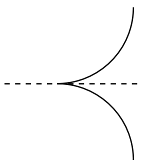
1. (a)



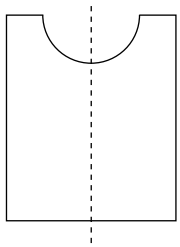
(b)



(c)



(d)



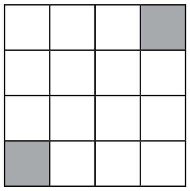
2.

	(i) Lines of symmetry	**(ii)** Order of rotational symmetry
(a)	2	2
(b)	0	4
(c)	8	8
(d)	3	3
(e)	1	1
(f)	0	2
(g)	2	4

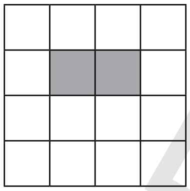
3. **(a)** 1
(b) 2
(c) 1
(d) 2
(e) 2
(f) 1
(g) 2
(h) 2

Challenge Yourself

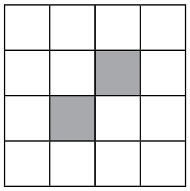
1. **(a)**



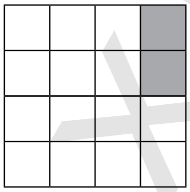
(b)



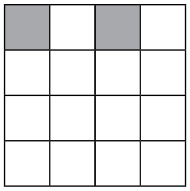
(c)



(d)



(e)



Teachers should note that there are other ways to shade the squares, and these are not the only answers.

Chapter 13 Statistical Data Handling

TEACHING NOTES

Suggested Approach

In primary school, students have learnt statistical diagrams such as pictograms, bar graphs, pie charts and line graphs. Here, students revisit what they have learnt and they are expected to know and appreciate the advantages and disadvantages of each diagram. With such knowledge, students can choose the most appropriate diagram given a certain situation. Teachers may want to give more examples when introducing the various stages of a statistical study and engage with students in evaluating and discussing the issues involved in each stage. Knowledge from past chapters may be required (i.e. percentage).

Section 13.1: Introduction to Statistics

Teachers should define statistics as the collection, organisation, display and interpretation of data. Teachers may want to briefly cover each stage of a statistical study and give real-life examples for discussion with students, in the later sections. Students are expected to solve problems involving various statistical diagrams.

Section 13.2: Pictograms and Bar Graphs

Using the example in the textbook, teachers can show how each stage is involved in a statistical study, where the data is displayed in the form of a pictogram and bar graph. Students should appreciate what happens in each stage, cumulating in the conclusion through the interpretation of the data. Through the example, students should also learn to read, interpret and solve problems using information presented in these statistical diagrams.

Students should know the characteristics of pictograms and bar graphs and take note of the merits and limitations of pictograms and bar graphs (see Attention on page 220 and Thinking Time on page 221).

Section 13.3: Pie Charts

Some students may still be unfamiliar with calculating the size of the angle of each sector in a pie chart. As such, teachers may wish to illustrate how this is done. Students need to recall the characteristics of a pie chart (see Attention on page 226).

Other than the examples given in the textbooks, teachers may give more examples where a data set is represented by a pie chart, such as students' views on recent current affairs.

Section 13.4: Line Graphs

Teachers may want to recap how line graphs are drawn. Students need to know the advantage, disadvantage and the cases line graphs are best used in. (see Attention on page 229).

Teachers can discuss some situations where pictograms, bar graphs, pie charts or line graphs are most suitable and assess students' understanding of statistical diagrams (see Class Discussion: Comparison of Various Statistical Diagrams).

Section 13.5: Statistics in Real-World Contexts

Teachers can use the examples given in the textbooks and further illustrate in detail how each stage in a statistical study is carried out using real-life examples.

Teachers can get the students to discuss and think of more ways to collect data besides conducting questionnaires. Other ways can include telephone interviews, emails, online surveys etc.

Teachers may want to assign small-scale projects for students where they conduct their own statistical studies. Such projects allow students to apply what they have learnt about statistical data handling in real-world contexts.

Section 13.6: Evaluation of Statistics

Teachers should go through the various examples in the textbook and discuss with students the potential issues that can arise at each stage of a statistical study. The importance of not engaging in any unethical behaviors, ensuring objectivity and providing the complete picture without omitting any forms of misrepresentation need to be inculcated into students.

WORKED SOLUTIONS

Thinking Time (Page 221)

1. Michael is correct. In a pictogram, each icon represents the same number. Hence, since there are 3 buses and 4 cars, more students travel to school by car than by bus.
2. To avoid a misinterpretation of the data, we can replace each bus and each car in the pictogram with a standard icon. Alternatively, we can draw the buses and the cars to be of the same size.

Class Discussion (Comparison of Various Statistical Diagrams)

1.

Statistical Diagram	Advantages	Disadvantages
Pictogram	• It is more colourful and appealing. • It is easy to read.	• It is difficult to use icons to represent exact values. • If the sizes of the icons are inconsistent, the data may easily be misinterpreted. • If the data has many categories, it is not desirable to use a pictogram to display it as it is quite tedious to draw so many icons.
Bar graph	• The data sets with the lowest and the highest frequencies can be easily identified. • It can be used to compare data across many categories. • Two or more sets of data with many categories can be easily compared.	• If the frequency axis does not start from 0, the displayed data may be misleading. • The categories can be rearranged to highlight certain results.
Pie chart	• The relative size of each data set in proportion to the entire set of data can be easily observed. • It can be used to display data with many categories. • It is visually appealing.	• The exact numerical value of each data set cannot be determined directly. • The sum of the angles of all the sectors may not be 360° due to rounding errors in the calculation of the individual angles. • It is not easy to compare across the categories of two or more sets of data.
Line graph	• Intermediate values can be easily obtained. • It can better display trends over time as compared to most of the other graphs. • The trends of two or more sets of data can be easily compared.	• Intermediate values may not be meaningful. • If the frequency axis does not start from 0, the displayed data may be misleading. • It is less visually appealing as compared to most of the other graphs.

2. **(a)** A bar graph should be used to display the data as we need to compare data across 12 categories. The categories with the lowest and the highest frequencies can also be easily identified.

(b) A line graph should be used to display the data as we need to display the trend of the change in the population of Singapore from the year 2004 to the year 2013.

(c) A pie chart cannot be used to display the data as we will not be able to directly determine the exact number of Secondary 1 students who travel to school by each of the 4 modes of transport. A line graph is inappropriate as it is used to display trends over time. Hence, a pictogram or a bar graph should be used to display the data. Since there are only 4 categories, we may wish to use a pictogram instead of a bar graph as it is more visually appealing and is easier to read.

(d) A pie chart should be used to display the data as it is easier to compare the relative proportions of Secondary 1 students who prefer the different drinks.

Performance Task (Page 232)

1. **Collection of Data**
   Guiding Questions:
   - What are the types of food that are sold in your current school canteen?
   - What other types of food would students like to be sold in the school canteen? How many choices would you like to include in the questionnaire?
   - What should be the sample size? How do you ensure that the sample chosen is representative of the entire school?
   - How many choices would you like each student surveyed to select?
2. **Organisation of Data**
   Guiding Questions:
   - How can you consolidate the data collected and present it in a table?
   - How should you organise the data such that it is easy to understand?
3. **Display of Data**
   Guiding Question:
   - Which statistical diagram, i.e. pictogram, bar graph, pie chart or line graph, is the most suitable to display the data obtained?
4. **Interpretation of Data**
   Guiding Questions:
   - How many more food stalls can your school canteen accommodate?
   - What is the conclusion of your survey, i.e. based on the statistical diagram drawn, which types of food stalls should your school engage for the school canteen?

Teachers may wish to refer students to pages 380 and 381 of the textbook for an example on how they can present their report.

Class Discussion (Evaluation of Statistics)

Part I: Collection of Data

1. *Teachers to conduct poll to find out the number of students who know Zidane, Beckenbauer and Cruyff. It is most likely that some students will know who Zidane is, but most (if not all) students will not know who Beckenbauer and Cruyff are.*
2. It is stated in the article that the poll was conducted on the UEFA website. As such, the voters who took part in the poll were most likely to belong to the younger generation who are more computer-savvy and hence, the voters were unlikely to be representative of all football fans.
3. As shown in the article, the number of votes for the three footballers were close, with 123 582 votes for Zidane, 122 569 votes for Beckenbauer and 119 332 votes for Cruyff. This is despite the fact that most of the younger generation, who were most likely to have voted in the poll, may not know who Beckenbauer and Cruyff are as they were at the peak of their careers in the 1970s. Hence, if older football fans were to participate in the poll, Zidane would probably not have come in first place.
4. The choice of a sample is important as if the sample chosen for collection of data is not representative of the whole population, the figures that are obtained may be misleading. Hence, a representative sample should be chosen whenever possible.

Part II: Organisation of Data

1. Banks and insurance firms, timeshare companies and motor vehicle companies received the most number of complaints.
2. The article states that banks and insurance firms, which were grouped together, received the most number of complaints. If banks and insurance firms were not grouped together, it is possible that timeshare companies received the most number of companies. For example, if the 1416 complaints were split equally between banks and insurance firms, they would have received 708 complaints each, then the number of complaints received by timeshare companies, i.e. 1238 complaints, would have been the greatest.
3. This shows that when organising data, it is important to consider whether to group separate entities as doing so might mislead consumers and result in inaccurate conclusions.

Part III: Display of Data

1. Although the height of the bar for Company E appears to be twice that of the bar for Company C, Company E's claim is not valid as the bars do not start from 0. By reading off the bar graph, Company E sold 160 light bulbs in a week, which is not twice as many as the 130 light bulbs sold by Company C in a week.
2. For bar graphs, if the vertical axis does not start from 0, the height of each bar will not be proportional to its corresponding frequency, i.e. number of light bulbs sold by each company in a week. Such display of statistical data may mislead consumers.

Part IV: Interpretation of Data

1. The conclusion was obtained based on a simple majority, i.e. since more than 50% of the employees were satisfied with working in the company, the survey concluded that the employees were satisfied with the company and that the company was a good place to work in.
2. $40\% \times 300 = \frac{40}{100} \times 300$
   $= 120$ employees

   It is stated in the article that 40% of the employees, i.e. 120 employees were not satisfied with working in the company. As such, even though a simple majority of the employees was satisfied with working in the company, it cannot be concluded that most of the employees were satisfied. This shows that we should not use simple majorities to arrive at conclusions or make decisions.
3. The amendment of the constitution of a country is a very serious matter where the agreement of a simple majority is insufficient, therefore there is a need for a greater percentage of elected Members of Parliament (MPs) to agree before the constitution can be amended. As a result, the Singapore government requires the agreement of at least a two-third majority before the constitution can be amended.

   Teachers may wish to take this opportunity to get students to search on the Internet for some laws that have been passed in the Singapore Parliament that resulted in a constitutional amendment.
4. It is important to have a basis or contention in order to decide on an issue, and that in some occasions, it is insufficient to make decisions based on a simple majority.

   Teachers may wish to ask students whether a simple majority, i.e. more than 50% of the votes, is necessary to decide on an issue. For example, in the 2011 Singapore Presidential Elections, Dr Tony Tan was elected President of the Republic of Singapore with 35.2% of the total valid votes cast.

Part V: Ethical Issues

It is unethical to use statistics to mislead others as it is essentially a form of misrepresentation and people may arrive at the wrong conclusions or make the wrong decisions.

The rationale for teaching students to be aware of how statistics can be used to mislead others is so that the students will be more discerning when they encounter statistics and will not be misled by others. Teachers should also impress upon students that they should not use statistics to mislead others because it is unethical to do so.

Practise Now 1

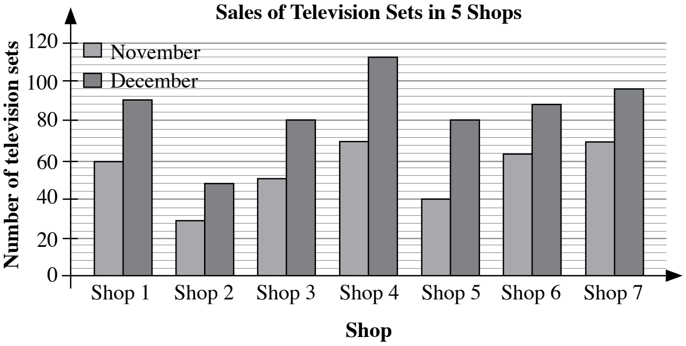
1. **(a)** **(i)** Profit earned by the company in 2010
   $= 5.5 \times$ PKR 1 000 000
   $=$ PKR 5 500 000

   (ii) Profit earned by the company in 2012
   $= 7 \times$ PKR 1 000 000
   $=$ PKR 7 000 000

(b) The company earned the least profit in 2009. The profit decreased by 1.5 × PKR 1 000 000 = PKR 1 500 000 in 2009 as compared to 2008.

Practise Now 2

1. (a)



(b) (i) Total number of television sets sold in the seven shops in November
$= 60 + 30 + 50 + 70 + 40 + 64 + 70$
$= 384$

(ii) Total number of television sets sold in the seven shops in December
$= 90 + 48 + 80 + 112 + 80 + 88 + 96$
$= 594$

(c) Required percentage $= \frac{384}{384 + 594} \times 100\%$
$= \frac{384}{978} \times 100\%$
$= 39\frac{43}{163}\%$

(d) (i) Required percentage $= \frac{70 + 96}{978} \times 100\%$
$= \frac{166}{978} \times 100\%$
$= 16\frac{476}{489}\%$

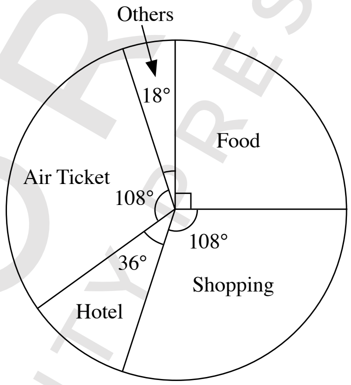
(ii) No, I do not agree with the manager. Since Shop 2 sold the least number of televsion sets in November and December, it should be closed down.

(e) The company performed better in terms of sales in December. This could be due to the fact that Christmas is in December when people buy television sets as gifts for others.

Practise Now 3

Farhan's total expenditure on the holiday
= PKR 1000 + PKR 1200 + PKR 400 + PKR 1200 + PKR 200
= PKR 4000

Item	Angle of sector
Food	$\frac{\text{PKR } 1000}{\text{PKR } 4000} \times 360° = 90°$
Shopping	$\frac{\text{PKR } 1200}{\text{PKR } 4000} \times 360° = 108°$
Hotel	$\frac{\text{PKR } 400}{\text{PKR } 4000} \times 360° = 36°$
Air Ticket	$\frac{\text{PKR } 1200}{\text{PKR } 4000} \times 360° = 108°$
Others	$\frac{\text{PKR } 200}{\text{PKR } 4000} \times 360° = 18°$



Practise Now 4

1. (i) $4x° + 2x° + 237.6° = 360°$ (∠s at a point)
$4x° + 2x° = 360° - 237.6°$
$6x° = 122.4°$
$x° = 20.4°$
$\therefore x = 20.4$

(ii) Required percentage $= \frac{4(20.4Υ)}{360Υ} \times 100\%$
$= \frac{81.6Υ}{360Υ} \times 100\%$
$= 22\frac{2}{3}\%$

(iii) Amount of fruit punch in the jar $= \frac{360Υ}{237.6Υ} \times 759$ ml
$= 1150$ ml

2. **(i)** The least popular colour is black.

(ii) Total number of cars sold

$= 2000 + 3500 + 5000 + 6000 + 1500$

$= 18\ 000$

Angle of sector that represents number of blue cars sold

$= \dfrac{2000}{18\ 000} \times 360°$

$= 40°$

Angle of sector that represents number of grey cars sold

$= \dfrac{3500}{18\ 000} \times 360°$

$= 70°$

Angle of sector that represents number of white cars sold

$= \dfrac{5000}{18\ 000} \times 360°$

$= 100°$

Angle of sector that represents number of red cars sold

$= \dfrac{6000}{18\ 000} \times 360°$

$= 120°$

Angle of sector that represents number of black cars sold

$= \dfrac{1500}{18\ 000} \times 360°$

$= 30°$

(iii) No, I do not agree with her. This is because the number of cars indicated on the *y*-axis is in thousands, thus 3500 grey cars and 1500 black cars are sold.

Practise Now 5

(i) The number of fatal road casualties was the highest in 2008.

(ii)

Year	2005	2006	2007	2008	2009
Number of fatal road casualties	173	190	214	221	183

(iii) Percentage decrease in number of fatal road casualties from 2008 to 2009

$= \dfrac{221 - 183}{221} \times 100\%$

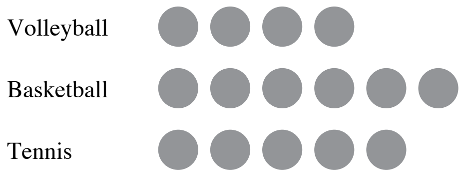
$= \dfrac{38}{221} \times 100\%$

$= 17\dfrac{43}{221}\%$

(iv) There are traffic cameras installed along more roads.

Exercise 13A

2. **(i)** **Students who Play Volleyball, Basketball or Tennis**



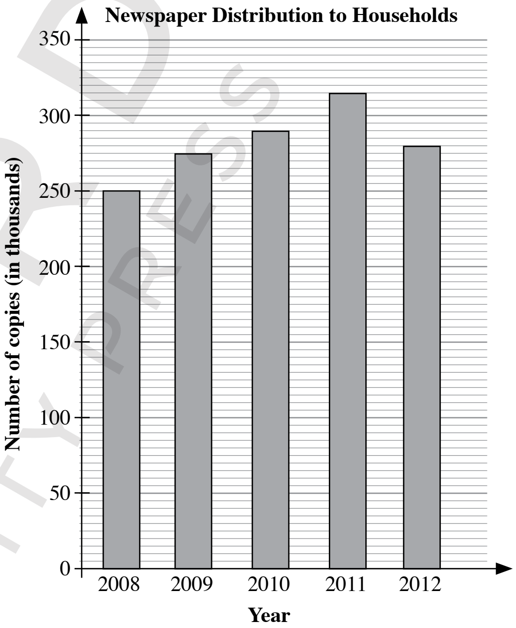
Each circle represents 10 students.

(ii) Required ratio = 4 : 5

(iii) Required percentage $= \dfrac{5}{6} \times 100\%$

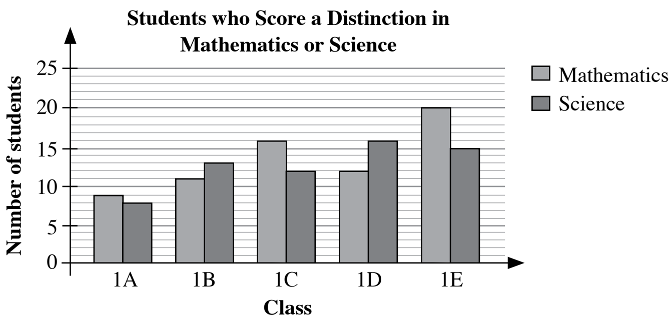
$= 83\dfrac{1}{3}\%$

3.



4. **(a)**

Class	Class 1A	Class 1B	Class 1C	Class 1D	Class 1E
Number of students who score a distinction in Mathematics	9	11	16	12	20
Number of students who score a distinction in Science	8	13	12	16	15



(b) (i) Total number of students in the 5 classes who score a distinction in Mathematics
$= 9 + 11 + 16 + 12 + 20$
$= 68$

(ii) Total number of students in the 5 classes who score a distinction in Science
$= 8 + 13 + 12 + 16 + 15$
$= 64$

(c) Required percentage $= \frac{12}{68} \times 100\%$
$= 17\frac{11}{17}\%$

(d) Percentage of students in Class 1D who score a distinction in Science
$= \frac{16}{40} \times 100\%$
$= 40\%$

(e) No, Jun Wei is not correct to say that there are 35 students in Class 1E. There may be students in the class who do not score distinctions in both Mathematics and Science. There may also be students in the class who score distinctions in both Mathematics and Science.

5. (i) Number of candidates who sat for the examination in 2009
$= 950$

(ii) Number of candidates who failed the examination in 2012
$= 500$

(iii) Total number of candidates who failed the examination in the six years
$= 400 + 350 + 350 + 400 + 450 + 500$
$= 2450$

$\therefore$ Required percentage $= \frac{500}{2450} \times 100\%$
$= 20\frac{20}{49}\%$

(iv) The percentage of successful candidates increases over the six years as they practise past-year papers and learn from their mistakes.

6. (i) Total number of workers employed in the housing estate
$= 4 \times 1 + 6 \times 2 + 5 \times 3 + 3 \times 4 + 2 \times 5$
$= 4 + 12 + 15 + 12 + 10$
$= 53$

(ii) Total number of shops in the housing estate
$= 4 + 6 + 5 + 3 + 2$
$= 20$
Number of shops hiring 3 or more workers $= 5 + 3 + 2$
$= 10$

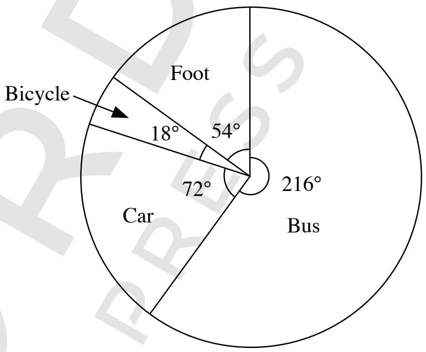
$\therefore$ Required percentage $= \frac{10}{20} \times 100\%$
$= 50\%$

(iii) Some shops have more customers as they are located at places with higher human traffic, thus they need to employ more workers.

Exercise 13B

1. Total number of students surveyed $= 768 + 256 + 64 + 192$
$= 1280$

Mode of transport	Angle of sector
Bus	$\frac{768}{1280} \times 360° = 216°$
Car	$\frac{256}{1280} \times 360° = 72°$
Bicycle	$\frac{64}{1280} \times 360° = 18°$
Foot	$\frac{192}{1280} \times 360° = 54°$



2. (i) Angle of sector that represents number of students who prefer yam $= 90°$

(ii) Angle of sector that represents number of students who prefer vanilla
$= 360° - 120° - 90° - 50°$ ($\angle$s at a point)
$= 100°$

(iii) Required percentage $= \frac{100\Upsilon}{360\Upsilon} \times 100\%$
$= 27\frac{7}{9}\%$

(iv) Total number of students in the class $= \frac{360\Upsilon}{50\Upsilon} \times 5$
$= 36$

3. (i) Required percentage $= \frac{180\Upsilon}{360\Upsilon} \times 100\%$
$= 50\%$

(ii) Required percentage $= \frac{72\Upsilon}{360\Upsilon} \times 100\%$
$= 20\%$

(iii) $x° = \frac{17\frac{1}{2}}{100} \times 360°$
$= 63°$
$\therefore x = 63$

4. **(i)** Total number of cars in the survey = 20 + 25 + 20 + 30 + 25
= 120

(ii) Total number of people in all the cars
= 20 × 1 + 25 × 2 + 20 × 3 + 30 × 4 + 25 × 5
= 20 + 50 + 60 + 120 + 125
= 375

(iii) Number of cars with 4 or more people = 30 + 25
= 55

$\therefore$ Required percentage = $\frac{55}{120} \times 100\%$

$= 45\frac{5}{6}\%$

(iv) Angle of sector that represents number of cars with 1 people

$= \frac{20}{120} \times 360°$

$= 60°$

Angle of sector that represents number of cars with 2 people

$= \frac{25}{120} \times 360°$

$= 75°$

Angle of sector that represents number of cars with 3 people

$= \frac{20}{120} \times 360°$

$= 60°$

Angle of sector that represents number of cars with 4 people

$= \frac{30}{120} \times 360°$

$= 90°$

Angle of sector that represents number of cars with 5 people

$= \frac{25}{120} \times 360°$

$= 75°$

5. **(i)**

Month	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Mass (kg)	3.2	3.4	3.8	4	4.2	4.4	5

(ii) Percentage increase in mass of the baby from the 4^{th} to 6^{th} month

$= \frac{5 - 4.2}{4.2} \times 100\%$

$= \frac{0.8}{4.2} \times 100\%$

$= 19\frac{1}{21}\%$

6. **(a)** Total angle of sectors that represent number of female students and teachers in the school
= 360° – 240° (∠s at a point)
= 120°

Angle of sector that represents number of teachers in the school

$= \frac{1}{6} \times 120°$

$= 20°$

(b) **(i)** Number of female students in the school = 5 × 45
= 225

(ii) Number of male students in the school = $\frac{240\Upsilon}{20\Upsilon} \times 45$
= 540

(c) Total school population = 45 + 225 + 540
= 810

Number of female teachers in the school = $\frac{2}{3} \times 45$
= 30

Number of females in the school = 225 + 30
= 255

$\therefore$ Required percentage = $\frac{255}{810} \times 100\%$

$= 31\frac{13}{27}\%$

7. $\frac{5}{1 + x + 5} \times 360° = 120°$

$$\frac{5}{6 + x} = \frac{120\Upsilon}{360\Upsilon}$$

$$\frac{5}{6 + x} = \frac{1}{3}$$

$$15 = 6 + x$$

$$\therefore x = 9$$

8. **(i)** The town had the greatest increase in the number of people from 2011 to 2012.

(ii)

Year	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Number of people (in thousands)	8	6	9	9.5	12	14	15	16	18	19	25

(iii) Percentage increase in number of people in the town from 2009 to 2012

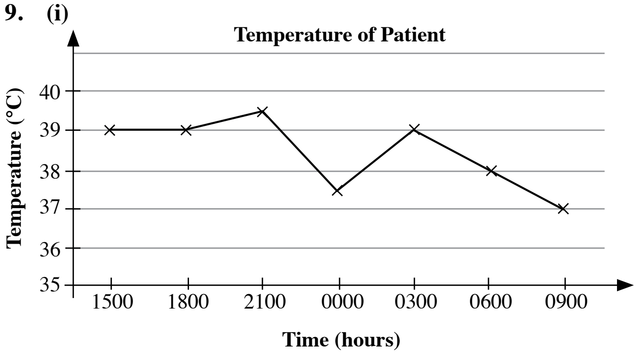
$= \frac{25\,000 - 16\,000}{16\,000} \times 100\%$

$= \frac{9000}{16\,000} \times 100\%$

$= 56\frac{1}{4}\%$

(iv) There are more new immigrants in the town.

9. **(i)**



(ii) Temperature of the patient at 1700 hours ≈ 39 °C
Temperature of the patient at 0100 hours ≈ 38 °C

10. The majority of the respondents in Kiran's survey are most likely females while those in Hussain's survey are most likely males. Kiran and Hussain may have conducted each of their surveys at a different location, e.g. Kiran may have conducted her survey at Orchard Road while Hussain may have conducted his survey at a housing estate.

11. No, I do not agree with Nora. The temperatures in both countries range from 24 °C to 35 °C. The temperatures in Country X seem to change more drastically than those in Country Y because the vertical axis of the line graph which shows the temperatures of Country X starts from 23 °C instead of 0 °C.

12. **(i)** Based on the 3-dimensional pie chart, Saad spends the most on luxury goods.

(ii) Based on the 2-dimensional pie chart, Saad spends the most on rent and luxury goods.

(iii) In a 3-dimensional pie chart, the sizes of the sectors will look distorted. The sectors towards the back of the pie chart will appear smaller than those towards the front.

13. No, I do not agree with Amirah. As there are more cars than motorcycles in Singapore, it is not surprising that there are more accidents involving cars than motorcycles. Moreover, there may be a higher chance of accidents involving motorcycles occurring due to the nature of the vehicle.

Review Exercise 13

1. **(i)** Required ratio $= 6 : 3$
$= 2 : 1$

(ii) Required percentage $= \frac{7}{4} \times 100\%$
$= 175\%$

2. **(i)** Total number of books read by the students in the class in a month
$= 2 \times 0 + 5 \times 1 + 9 \times 2 + 8 \times 3 + 6 \times 4 + 5 \times 5 + 1 \times 6$
$= 0 + 5 + 18 + 24 + 24 + 25 + 6$
$= 102$

(ii) Number of students who read more than 4 books $= 5 + 1$
$= 6$

Total number of students in the class $= 2 + 5 + 9 + 8 + 6 + 5 + 1$
$= 36$

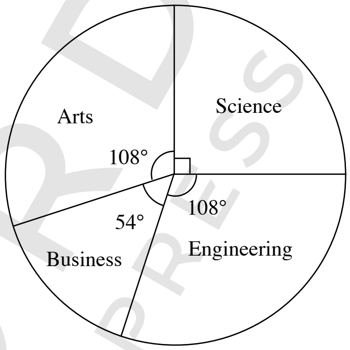
$\therefore$ Required percentage $= \frac{6}{36} \times 100\%$
$= 16\frac{2}{3}\%$

(iii) Number of students who read fewer than 3 books $= 2 + 5 + 9$
$= 16$

Angle of sector that represents number of students who read fewer than 3 books
$= \frac{16}{36} \times 360°$
$= 160°$

3. Percentage of students who are enrolled in the Arts course
$= 100\% - 25\% - 30\% - 15\%$
$= 30\%$

Type of course	Angle of sector
Science	$\frac{25}{100} \times 360° = 90°$
Engineering	$\frac{30}{100} \times 360° = 108°$
Business	$\frac{15}{100} \times 360° = 54°$
Arts	$\frac{30}{100} \times 360° = 108°$



4. **(i)** Total angle of sectors that represent amount Bina spends on clothes and food
$= 360° - 36° - 90° - 90°$ ($\angle$s at a point)
$= 144°$
Angle of sector that represents amount Bina spends on food
$= \frac{1}{4} \times 144°$
$= 36°$

$\therefore$ Required percentage $= \frac{36\Upsilon}{90\Upsilon} \times 100\%$
$= 40\%$

(ii) Bina's monthly income $= \frac{360\Upsilon}{36\Upsilon} \times$ PKR 400
= PKR 4000

Bina's annual income = 12 × PKR 4000
= PKR 48 000

5. (i)

Year	2008	2009	2010	2011	2012
Number of laptops	70	30	44	90	26

(ii) Percentage decrease in number of laptops purchased by the company from 2008 to 2009

$= \frac{70 - 30}{70} \times 100\%$

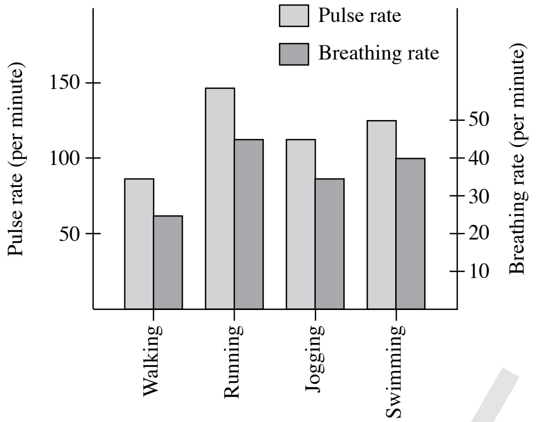
$= \frac{40}{70} \times 100\%$

$= 57\frac{1}{7}\%$

(iii) The company might have had a tighter budget in 2009.

Challenge Yourself

The better way to display the data using a bar graph is as follows:



Chapter 14 Averages of Statistical Data

TEACHING NOTES

Suggested Approach

Students will learn that the average of a set of data is the sum of all the data divided by the number of data. Teachers can further explain that in statistics, there are other types of 'averages'. The average that students will learn is also known as the mean. In this chapter, students are to know and learn the properties of median and mode as well.

By the end of the chapter, students should know how to calculate mean, median and mode from the various statistical diagrams and be aware of the situations where one numerical measure is preferred over another.

Section 14.2: Mean

Teachers can guide students through the worked examples to show how the mean is calculated. Students should be reminded to be careful not to miss out any values or use any wrong values in the calculation.

Teachers should note calculating the mean from a frequency table as well as estimating the mean of a set of grouped data are new to students. More practice and guidance may be required for some students here.

Section 14.3: Median

The definition and purpose of a median should be well-explained to the students. The example on page 248 is a good example why the median is preferred over the mean. Students may need to be reminded that the numerical average is to give the best representation of any data.

The main features of finding the median, namely whether the number of data is even or odd and that the data must be arranged in order, are important and must be emphasised to students. The activities are meant to test and reinforce students' understanding (see Thinking Time on page 250 and Class Discussion: Creating Sets of Data with Given Conditions).

Section 14.4: Mode

The mode is arguably the easiest numerical average that students will need to learn, as it involves identifying the most frequent data without any calculations involved. Teachers ought to be able to quickly go through the examples of finding the mode from the various statistical diagrams.

Section 14.5 Mean, Median and Mode

Questions involving all three numerical averages will be covered in this section. Students may need to recall the algebraic skills they have picked up at the first half of the textbook.

In this section, teachers should use the activities that compare the mean, median mode and question students on the most suitable numerical average depending on the set of data provided (see Thinking Time on page 256 and Class Discussion: Comparison of Mean, Median and Mode).

WORKED SOLUTIONS

Thinking Time (Page 250)

Rearranging the data in descending order instead, we have,
30, 21, 19, 14, 12, 9, 8, 5

$\therefore$ Median = mean of the data in the 4^{th} and the 5^{th} position

$$= \frac{14 + 12}{2}$$

$$= 13$$

Hence, the median remains the same if the data is arranged in descending order instead.

Class Discussion (Creating Sets of Data with Given Conditions)

Some sets of data are shown as follows.

(i) 1, 1, 1, 1, 2, 4, 11
Difference between minimum and maximum value = 10
Mean = 3
Median = 1

(ii) 1, 1, 1, 2, 2, 3, 11
Difference between minimum and maximum value = 10
Mean = 3
Median = 2

(iii) 1, 1, 1, 1, 5, 8, 11
Difference between minimum and maximum value = 10
Mean = 4
Median = 1

(iv) 1, 1, 2, 2, 2, 9, 11
Difference between minimum and maximum value = 10
Mean = 4
Median = 2

(v) 1, 2, 3, 4, 5, 9, 11
Difference between minimum and maximum value = 10
Mean = 5
Median = 4

Teachers may wish to note the sets of data are not exhaustive. To come up with a set of data, it is recommended that the mean and median are decided, before working backwards.

Thinking Time (Page 253)

Some sets of data are shown as follows.

(i) 41, 56, 56, 58, 59, 60
Mean = 55
Mode = 56
Median = 57

(ii) 39, 56, 56, 57, 58, 59, 60
Mean = 55
Mode = 56
Median = 57

(ii) 35, 55, 56, 56, 58, 59, 60, 61
Mean = 55
Mode = 56
Median = 57

(iv) 36, 57, 57, 59, 60, 61
Mean = 55
Mode = 57
Median = 58

(v) 27, 58, 58, 59, 60, 61, 62
Mean = 55
Mode = 58
Median = 59

Teachers may wish to note the sets of data are not exhaustive. To come up with a set of data, it is recommended that the mean, mode and median are decided, before working backwards. Note that the number of data is not specified.

Thinking Time (Page 256)

$$\text{Mean monthly salary} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{12 \times 15\,000 + 5 \times 50\,000 + 2 \times 100\,000 + 4 \times 150\,000 + 1 \times 250\,000 \times 1 \text{ x } 500\,000}{25}$$

$$= \frac{1\,980\,000}{25}$$

$$= \text{PKR } 79\,200$$

The average monthly salary of the employees is PKR 79 200 refers to the mean monthly salary of the employees. The average monthly salary can mean the median monthly salary, which is PKR 50 000 or the modal monthly salary, which is PKR 15 000, as well.

Hence, Bina's statement does not give a good picture of how much the employees earn.

Number of employees who earn PKR 15 000 = 12

Percentage of employees who earn PKR 15 000 $= \frac{12}{25} \times 100\%$
$= 48\% \approx 50\%$

Faiza's statement that almost half of the employees earn PKR 15 000 is correct but it does not state the amount the other employees in the company earn.

Number of employees who earn at least PKR 50 000 = 13

Percentage of employees who earn at least PKR 50 000 $= \frac{13}{25} \times 100\%$
$= 52\% > 50\%$

The statement that more than 50% of the employees earn at least PKR 50 000
is correct. It also gives the best picture of how much money the employees earn in the company, since the statement allows us to infer the amount the rest of the employees earn.

Hussain's statement gives the best picture of how much the employees in the company earn.

Class Discussion (Comparison of Mean, Median and Mode)

1. **(i)** Mean $= \dfrac{15 + 17 + 13 + 18 + 20 + 19 + 15}{7}$

$= \dfrac{117}{7}$

$= 16\dfrac{5}{7}$

Total number of data = 7

Middle position $= \dfrac{7 + 1}{2}$

$= 4^{\text{th}}$ position

Rearranging the data in ascending order,

13, 15, 15, 17, 18, 19, 20

$\therefore$ Median = data in the 4^{th} position

$= 17$

Mode = 15

(ii) The mean and median will change while the mode will remain the same.

New mean $= \dfrac{117 + 55}{7 + 1}$

$= \dfrac{172}{8}$

$= 21\dfrac{1}{2}$

Total number of data = 7

Middle position $= \dfrac{8 + 1}{2}$

$= 4.5^{\text{th}}$ position

Rearranging the data in ascending order,

13, 15, 15, 17, 18, 19, 20, 55

$\therefore$ New median = mean of the data in the 4^{th} position and 5^{th} position

$= \dfrac{17 + 18}{2}$

$= 17.5$

New mode = 15

(iii) The mean is most affected by the addition of a large number.

It had the biggest difference of $21\dfrac{1}{2} - 16\dfrac{5}{7} = 4\dfrac{11}{14}$.

(iv) The mean will be most affected by extreme values.

The mode will remain unchanged by extreme values.

Hence, the median is the most appropriate measure to use.

2. **(i)** Mean $= \dfrac{6 + 7 + 8 + 8 + 7 + 9 + 5 + 6 + 6}{9}$

$= \dfrac{62}{9}$

$= 6\dfrac{8}{9}$

Total number of data = 9

Middle position $= \dfrac{9 + 1}{2}$

$= 5^{\text{th}}$ position

Rearranging the data in ascending order,

5, 6, 6, 6, 7, 7, 8, 8, 9

$\therefore$ Median = data in the 5^{th} position

$= 7$

Mode = 6

(ii) The mode best represents the sizes of shoes sold because it represents the size of the shoes most sold.

3. **(i)** Mean $= \dfrac{2 + 3 + 1 + 4 + 5 + 1 + 2 + 2 + 1 + 1}{10}$

$= \dfrac{22}{10}$

$= 2.2$

Total number of data = 10

Middle position $= \dfrac{10 + 1}{2}$

$= 5.5^{\text{th}}$ position

Rearranging the data in ascending order,

1, 1, 1, 1, 2, 2, 2, 3, 4, 5

$\therefore$ Median = mean of the data in the 5^{th} and the 6^{th} position

$= \dfrac{2 + 2}{2}$

$= 2$

Mode = 1

(ii) Even though the mean is not an integer, it still has a physical meaning.

i.e. 2.2 children per family is equivalent to <u>22</u> children in 10 families.

4. The mean is preferred when there are no extreme values in the set of data. Comparatively, the median is preferred when there are extreme values.

The mode is preferred when we want to know the most common value in a data set.

Practise Now 1

Mean score $= \dfrac{\text{Sum of scores}}{\text{Number of students}}$

$= \dfrac{79 + 58 + 73 + 66 + 50 + 89 + 91 + 58}{8}$

$= \dfrac{564}{8}$

$= 70.5$

Practise Now 2

$$\frac{44 + 47 + y + 58 + 55}{5} = 52$$

$$44 + 47 + y + 58 + 55 = 260$$

$$204 + y = 260$$

$$\therefore y = 56$$

Practise Now 3

1. (i) Since mean $= \dfrac{\text{sum of the 7 numbers}}{7}$,

then sum of the 7 numbers $= 7 \times \text{mean}$
$= 7 \times 11$
$= 77$

(ii) $3 + 17 + 20 + 4 + 15 + y + y = 77$
$59 + 2y = 77$
$2y = 18$
$y = 9$

2. Since mean $= \dfrac{\text{sum of the heights of 20 boys and 14 girls}}{34}$

then sum of the heights of 20 boys and 14 girls $= 34 \times \text{mean}$
$= 34 \times 161$
$= 5474$ cm

Since mean $= \dfrac{\text{sum of the heights of 14 girls}}{14}$

then sum of the heights of 14 girls $= 14 \times \text{mean}$
$= 14 \times 151$
$= 2114$ cm

Sum of the heights of 20 boys $= 5474 - 2114$
$= 3360$ cm

Mean height of the 20 boys $= \dfrac{3360}{20}$
$= 168$ cm

3. $\dfrac{16 + w + 17 + 9 + x + 2 + y + 7 + z}{9} = 11$

$16 + w + 17 + 9 + x + 2 + y + 7 + z = 99$
$51 + w + x + y + z = 99$
$w + x + y + z = 48$

Mean of w, x, y and z $= \dfrac{48}{4}$
$= 12$

Practise Now 4

(i) Total number of visitors $= 12 + 32 + 54 + 68 + 18 + 16$
$= 200$

(ii) Total amount of money spent by the visitors
$= 12 \times$ PKR 40 $+ 32 \times$ PKR 60 $+ 54 \times$ PKR 80 $+ 68 \times$ PKR 100 $+ 18 \times$ PKR 160 $+ 16 \times$ PKR 200
= PKR 19 600

(iii) Mean amount of money spent by the visitors $= \dfrac{\text{PKR 19 600}}{200}$
= PKR 98

Practise Now 5

1. Mean number of siblings $= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{4 \times 0 + 5 \times 1 + 3 \times 2 + 2 \times 3 + 1 \times 4}{15}$

$= \dfrac{21}{15}$

$= 1.4$

2. Mean pH value of the solutions

$= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{1.4 + 1.4 + 1.9 + 2.3 + 2.6 + 2.7 + 2.7 + 2.7 + 2.8 + 3.3 + 3.6 + 3.6 + 3.6 + 3.9 + 4.1 + 4.2 + 4.2 + 4.5 + 4.5 + 4.9}{20}$

$= \dfrac{64.9}{20}$

$= 3.245$

Practise Now 6

(a) Total number of data = 7

Middle position $= \dfrac{7 + 1}{2}$
$= 4^{\text{th}}$ position

Rearranging the data in ascending order, we have:
3, 9, 11, 15, 16, 18 and 20

$\therefore$ Median = data in the 4^{th} position
$= 15$

(b) Total number of data = 5

Middle position $= \dfrac{5 + 1}{2}$
$= 3^{\text{rd}}$ position

Rearranging the data in ascending order, we have:
11.2, 15.6, 17.3, 18.2 and 30.2

$\therefore$ Median = data in the 3^{rd} position
$= 17.3$

Practise Now 7

(a) Total number of data = 6

Middle position $= \dfrac{6 + 1}{2}$
$= 3.5^{\text{th}}$ position

Rearranging the data in ascending order, we have:
12, 15, 15, 20, 25 and 32

$\therefore$ Median = mean of the data in the 3^{rd} and the 4^{th} position

$= \dfrac{15 + 20}{2}$

$= 17.5$

(b) Total number of data = 8

Middle position $= \dfrac{8 + 1}{2}$
$= 4.5^{\text{th}}$ position

Rearranging the data in ascending order, we have:
6.7, 6.8, 7.3, 8, 8.8, 8.9, 8.9, 10

$\therefore$ Median = mean of the data in the 4^{th} and the 5^{th} position

$= \dfrac{8 + 8.8}{2}$

$= 8.4$

Practise Now 8

1. Total number of data = 15

$$\text{Middle position} = \frac{15 + 1}{2}$$
$$= 8^{\text{th}} \text{ position}$$
$$\therefore \text{Median} = \text{data in the } 8^{\text{th}} \text{ position}$$
$$= 1$$

Practise Now 9

Total number of data = 28

$$\text{Middle position} = \frac{28 + 1}{2}$$
$$= 14.5^{\text{th}} \text{ position}$$
$$\therefore \text{Median time} = \text{mean of the data in the } 14^{\text{th}} \text{ and the } 15^{\text{th}} \text{ position}$$
$$= \frac{7 + 7}{2}$$
$$= 7 \text{ minutes}$$

Practise Now 10

(i) Modal lengths = 60 cm, 110 cm

(ii) Modal length = 60 cm

Practise Now 11

(a) Mode = 0.4

(b) Modes = 32, 37

(c) Mode = 1

(d) Mode = PKR 3000

Practise Now 12

(a)
$$\frac{2 \times 0 + x \times 1 + 3 \times 2 + 4 \times 3 + 1 \times 4}{2 + x + 3 + 4 + 1} = 1.8$$
$$\frac{x + 22}{x + 10} = 1.8$$
$$x + 22 = 1.8(x + 10)$$
$$x + 22 = 1.8x + 18$$
$$0.8x = 4$$
$$x = 5$$

(b) We write the data as follows:

$$0, 0, \underbrace{1, ..., 1}_{x}, 2, 2, 2, 3, 3, 3, 3, 4$$

The greatest value of x occurs when the median is here.

$$\therefore 2 + x = 2 + 4 + 1$$
$$2 + x = 7$$
$$x = 5$$
$$\therefore \text{Greatest value of } x = 5$$

The smallest value of x occurs when the median is here.

$$\therefore 2 + x + 2 = 4 + 1$$
$$4 + x = 5$$
$$x = 1$$
$$\therefore \text{Smallest value of } x = 1$$

$$\therefore \text{Possible values of } x = 1, 2, 3, 4, 5$$

(c) Greatest possible value of $x = 3$

Exercise 14A

1. Mean number of passengers

$$= \frac{\text{Sum of number of passengers}}{\text{Number of coaches}}$$
$$= \frac{29 + 42 + 45 + 39 + 36 + 41 + 38 + 37 + 43 + 35 + 32 + 40}{12}$$
$$= \frac{457}{12}$$
$$= 38.1 \text{ (to 3 s.f.)}$$

2. Mean price of books

$$= \frac{\text{Sum of prices of books}}{\text{Number of books}}$$
$$= \frac{19.90 + 24.45 + 34.65 + 26.50 + 44.05 + 38.95 + 56.40 + 48.75 + 29.30 + 35.65}{10}$$
$$= \frac{358.6}{10}$$
$$= \text{PKR } 35.86$$

3.
$$\frac{7 + 15 + 12 + 5 + h + 13}{6} = 10$$
$$52 + h = 60$$
$$h = 8$$

4. Since mean of masses of 5 boys $= \dfrac{\text{sum of the masses of 5 boys}}{5}$

then sum of the mass of 5 boys $= 5 \times$ mean
$$= 5 \times 62$$
$$= 310 \text{ kg}$$

Since mean of masses of 4 boys $= \dfrac{\text{sum of the masses of 4 boys}}{4}$

then sum of the mass of 4 boys $= 4 \times$ mean
$$= 4 \times 64$$
$$= 256 \text{ kg}$$

Mass of boy excluded $= 310 - 256$
$$= 54 \text{ kg}$$

5. **(i)** Since mean of 8 numbers $= \dfrac{\text{sum of the 8 numbers}}{8}$,

then sum of the 8 numbers $= 8 \times$ mean
$$= 8 \times 12$$
$$= 96$$

(ii)
$$6 + 8 + 5 + 10 + 28 + k + k + k = 96$$
$$57 + 3k = 96$$
$$3k = 39$$
$$k = 13$$

6. **(i)** Total number of matches played $= 6 + 8 + 5 + 6 + 2 + 2 + 1$
$$= 30$$

(ii) Total number of goals scored
$$= 6 \times 0 + 8 \times 1 + 5 \times 2 + 6 \times 3 + 2 \times 4 + 2 \times 5 + 1 \times 6$$
$$= 60$$

(iii) Mean number of goals scored per match $= \dfrac{60}{30}$
$$= 2$$

7. Mean number of days of absence

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{23 \times 0 + 4 \times 1 + 5 \times 2 + 2 \times 3 + 2 \times 4 + 1 \times 5 + 2 \times 6 + 1 \times 9}{23 + 4 + 5 + 2 + 2 + 1 + 2 + 1}$

$= \frac{54}{40}$

$= 1.35$ days

8. (a) Mean

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{1 \times 6 + 2 \times 7 + 1 \times 8 + 4 \times 9 + 3 \times 10 + 1 \times 11 + 1 \times 12}{1 + 2 + 1 + 4 + 3 + 1 + 1}$

$= \frac{117}{13}$

$= 9$

(b) Mean

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{7.2 + 7.3 + 7.5 + 7.5 + 8.2 + 8.7 + 8.8 + 8.8 + 8.9 + 8.9 + 8.9 + 9.1 + 9.3 + 9.7 + 9.7 + 10.2 + 10.7 + 10.8}{4 + 7 + 4 + 3}$

$= \frac{160.2}{18}$

= PKR 8.90

9. (i) Since mean of 10 numbers $= \frac{\text{sum of 10 numbers}}{10}$,

then sum of 10 numbers $= 10 \times \text{mean}$

$= 10 \times 14$

$= 140$

Since mean of 3 numbers $= \frac{\text{sum of 3 numbers}}{3}$,

then sum of 3 numbers $= 3 \times \text{mean}$

$= 3 \times 4$

$= 12$

Sum of the remaining seven numbers $= 140 - 12$

$= 128$

(ii) $15 + 18 + 21 + 5 + m + 34 + 14 = 128$

$107 + m = 128$

$m = 21$

10. Since mean monthly wage of 12 workers $= \frac{\text{sum of monthly wages}}{12}$,

then sum of monthly wages $= 12 \times \text{mean}$

$= 12 \times 1000$

= PKR 12 000

Since mean monthly wage of 5 inexperienced workers

$= \frac{\text{sum of monthly wages}}{5}$,

then sum of monthly wages $= 5 \times \text{mean}$

$= 5 \times 846$

= PKR 4230

Sum of monthly wages of 7 experienced workers = 12 000 – 4230

= PKR 7770

Mean monthly wage of 7 experienced workers $= \frac{7770}{7}$

= PKR 1110

11. (i) Since mean height $= \frac{\text{sum of heights of 3 plants}}{3}$,

then sum of heights of 3 plants $= 3 \times \text{mean}$

$= 3 \times 30$

$= 90$ cm

Height of plant $B = \frac{3}{2 + 3 + 5} \times 90$

$= \frac{3}{10} \times 90$

$= 27$ cm

(ii) Since mean height $= \frac{\text{sum of heights of 4 plants}}{4}$

then sum of heights of 4 plants $= 4 \times \text{mean}$

$= 4 \times 33$

$= 132$ cm

Height of plant $D = 132 - 90$

$= 42$ cm

12. (a) Mean mark of students for English

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{0 \times 0 + 1 \times 1 + 6 \times 2 + 14 \times 3 + 4 \times 4 + 8 \times 5 + 2 \times 6 + 4 \times 7 + 0 \times 8 + 1 \times 9 + 0 \times 10}{40}$

$= \frac{160}{40}$

$= 4$ marks

Mean mark of students for Mathematics

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{0 \times 0 + 4 \times 1 + 1 \times 2 + 6 \times 3 + 5 \times 4 + 10 \times 5 + 3 \times 6 + 5 \times 7 + 3 \times 8 + 1 \times 9 + 2 \times 10}{40}$

$= \frac{200}{40}$

$= 5$ marks

(b) (i) Passing mark for English $= 50\% \times 10$

$= \dfrac{50}{100} \times 10$

$= 5$

Number of students who passed English $= 8 + 2 + 4 + 1$

$= 15$

Percentage of students who passed English $= \dfrac{15}{40} \times 100\%$

$= 37\dfrac{1}{2}\%$

(ii) Passing mark for Mathematics $= 50\% \times 10$

$= \dfrac{50}{100} \times 100$

$= 5$

Number of students who did not pass Mathematics

$= 4 + 1 + 6 + 5$

$= 16$

Percentage of students who did not pass Mathematics

$= \dfrac{16}{40} \times 100\%$

$= 40\%$

13. Since mean of x, y and $z = \dfrac{\text{Sum of } x, y, \text{ and } z}{3}$,

then sum of x, y and $z = 3 \times$ mean

$= 3 \times 6$

$= 18$

Since mean of x, y, z, a and $b = \dfrac{\text{Sum of } x,\ y,\ z,\ a \text{ and } b}{5}$,

then sum of x, y, z, a and $b = 5 \times$ mean

$= 5 \times 8$

$= 40$

Sum of a and $b = 40 - 18$

$= 22$

Mean of a and $b = \dfrac{22}{2}$

$= 11$

Exercise 14B

1. (a) Total number of data = 7

Middle position $= \dfrac{7 + 1}{2}$

$= 4^{\text{th}}$ position

Rearranging the data in ascending order, we have:

1, 3, 5, 5, 5, 6, 6

$\therefore$ Median = data in the 4^{th} position

$= 5$

(b) Total number of data = 6

Middle position $= \dfrac{6 + 1}{2}$

$= 3.5^{\text{th}}$ position

Rearranging the data in ascending order, we have:

25, 28, 29, 30, 33, 37

$\therefore$ Median = mean of the data in the 3^{rd} and the 4^{th} position

$= \dfrac{29 + 30}{2}$

$= 29.5$

(c) Total number of data = 7

Middle position $= \dfrac{7 + 1}{2}$

$= 4^{\text{th}}$ position

Rearranging the data in ascending order, we have:

1.1, 1.2, 1.6, 2.8, 3.2, 4.1, 4.1

$\therefore$ Median = data in the 4^{th} position

$= 2.8$

(d) Total number of data = 8

Middle position $= \dfrac{8 + 1}{2}$

$= 4.5^{\text{th}}$ position

Rearranging the data in ascending order, we have:

4.7, 5.5, 8.4, 12, 13.5, 22.6, 31.3, 39.6

$\therefore$ Median = mean of the data in the 4^{th} and the 5^{th} position

$= \dfrac{12 + 13.5}{2}$

$= 12.75$

2. (a) Total number of data = 20

Middle position $= \dfrac{20 + 1}{2}$

$= 10.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 10^{th} and the 11^{th} position

$= \dfrac{39 + 40}{2}$

$= 39.5$

(b) Total number of data = 17

Middle position $= \dfrac{17 + 1}{2}$

$= 9^{\text{th}}$ position

$\therefore$ Median = data in the 11^{th} position

$= 5.7$

(c) Total number of data = 42

Middle position $= \dfrac{42 + 1}{2}$

$= 21.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 21^{th} and the 22^{th} position

$= \dfrac{40 + 45}{2}$

$= 42.5$

3. **(a)** Mode = 3

(b) Modes = 7.7, 9.3

4. **(a)** Mode = Red

(b) Modes = 78, 79

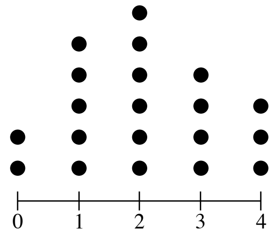
(c) Mode = 60

(d) Each value of x occurs only once. Hence, there is no mode.

5. **(i)** Modal temperature = 27 °C

(ii) Modal temperatures = 22 °C, 27 °C

6. **(a)**



(b) **(i)** $\text{Mean} = \dfrac{2 \times 0 + 5 \times 1 + 6 \times 2 + 4 \times 3 + 3 \times 4}{2 + 5 + 6 + 4 + 3}$

$= \dfrac{41}{20}$

$= 2.05$

(ii) Total number of data = 20

$\text{Middle position} = \dfrac{20 + 1}{2}$

$= 10.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 10^{th} and the 11^{th} position

$= \dfrac{2 + 2}{2}$

$= 2$

(iii) Mode = 2

7. **(a)** The lengths of the pendulums measured by group A range from 49 cm to 85 cm.

The lengths of the pendulums measured by group B range from 53 cm to 83 cm. The length is clustered around 65 cm to 67 cm.

(b) I disagree with the statement. The modal length for group A is 53 cm, and is shorter than the modal lengths for group B, which are 66 cm and 73 cm.

8. Let the eighth number be x.

Total number of data = 8

$\text{Middle position} = \dfrac{8 + 1}{2}$

$= 4.5^{\text{th}}$ position

Rearranging the numbers in ascending order,

we can have x, 1, 2, 3, 4, 9, 12, 13 or 1, 2, 3, 4, 9, 12, 13, x or x is between any 2 numbers.

The median is the mean of the 4^{th} position and 5^{th} position.

Since $\dfrac{3 + 4}{2} = 3.5 \neq 4.5$ and $\dfrac{4 + 9}{2} = 6.5 \neq 4.5$, x must be in the 4^{th} position or 5^{th} position.

If x is in the 4^{th} position, $\dfrac{x + 4}{2} = 4.5$

$x + 4 = 9$

$x = 5$ (rejected, since $x \leqslant 4$)

If x is in the 5^{th} position, $\dfrac{4 + x}{2} = 4.5$

$4 + x = 9$

$x = 5$

The eighth number is 5.

9. **(a)** **(i)** Mean distance

$$= \frac{23 + 24 + 26 + 29 + 30 + 31 + 32 + 32 + 32 + 34 + 34 + 35 + 38 + 42 + 42}{15}$$

$= \dfrac{484}{15}$

$= 32.3$ km (to 3 s.f.)

(ii) Total number of data = 15

$\text{Middle position} = \dfrac{15 + 1}{2}$

$= 8^{\text{th}}$ position

$\therefore$ Median distance = data in the 8^{th} position

$= 32$ km

(iii) Modal distance = 32 km

(b) There are 3 prime numbers, i.e. 23, 29 and 31.

$\text{P(distance covered is a prime number)} = \dfrac{3}{15}$

$= \dfrac{1}{5}$

10. **(a)** **(i)** $x + 2 + y + 6 + 14 = 40$

$x + y + 22 = 40$

$\therefore x + y = 18$ (shown)

(ii) Mean $= 64$

$\dfrac{x \times 2 + 2 \times 4 + y \times 6 + 6 \times 8 + 14 \times 10}{40} = 6.4$

$2x + 8 + 6y + 48 + 140 = 256$

$2x + 6y + 196 = 256$

$2x + 6y = 60$

$\therefore x + 3y = 30$ (shown)

(iii) $x + y = 18$ — (1)

$x + 3y = 30$ — (2)

(2) – (1):

$(x + 3y) - (x + y) = 30 - 18$

$x + 3y - x - y = 12$

$2y = 12$

$y = 6$

Substitute $y = 6$ into (1):

$x + 6 = 18$

$x = 12$

$\therefore\ x = 12, y = 6$

(b) (i) Total number of data = 40

Middle position = $\dfrac{40 + 1}{2}$

$= 20.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 20^{th} and the 21^{th} position

$= \dfrac{6 + 8}{2}$

$= 7$

(ii) Mode = 10

11. (a) We write the data as follows:

$\underbrace{0, ..., 0}_{5}, 1, 1, 2, \underbrace{3, ..., 3}_{x}$

Since the median is 2,

$\therefore\ 5 + 2 = x$

$x = 7$

(b) $\underbrace{0, ..., 0}_{5}, 1, 1, 2, \underbrace{3, ..., 3}_{x}$

The smallest value of x occurs when the median is here.	The greatest value of x occurs when the median is here.
$\therefore\ 5 = 1 + 1 + x$	$\therefore\ 5 + 1 = 1 + x$
$5 = 2 + x$	$6 = 1 + x$
$x = 3$	$x = 5$
$\therefore$ Smallest value of $x = 3$	$\therefore$ Greatest value of $x = 5$

$\therefore$ Possible values of $x = 3, 4, 5$

12. (i) Salman's mean score $= \dfrac{45}{9}$

$= 5$

Saad's mean score $= \dfrac{42}{9}$

$= 4.67$ (to 3 s.f.)

(ii) Salman scored better on most of the holes. The mean scores do not indicate this.

(iii) Total number of data = 9

Middle position = $\dfrac{9 + 1}{2}$

$= 5^{\text{th}}$ position

Rearranging Salman's scores in ascending order,

2, 2, 2, 3, 3, 4, 5, 7, 17

$\therefore$ Salman's median score = data in the 5^{th} position

$= 3$

Rearranging Saad's scores in ascending order,

2, 3, 3, 4, 4, 6, 6, 6, 8

$\therefore$ Saad's median score = data in the 5^{th} position

$= 4$

(iv) Salman's modal score = 2

Saad's modal score = 6

(v) The mode gives the best comparison. The mode shows the most common score of each player, thus demonstrates the ability of each player the best.

13. (a) $5 + 13 + 15 + x + 1 + y + 2 = 50$

$x + y + 36 = 50$

$x + y = 14$ — (1)

Mean $= 2.18$

$$\frac{5 \times 0 + 13 \times 1 + 15 \times 2 + x \times 3 + 1 \times 4 + y \times 5 + 2 \times 6}{50} = 2.18$$

$$\frac{3x + 5y + 59}{50} = 2.18$$

$3x + 5y + 59 = 109$

$3x + 5y = 50$ — (2)

$5 \times (1)$: $5x + 5y = 70$ — (3)

(3) – (2):

$(5x + 5y) - (3x + 5y) = 70 - 50$

$5x + 5y - 3x - 5y = 20$

$2x = 20$

$x = 10$

Substitute $x = 10$ into (1):

$10 + y = 14$

$y = 4$

$\therefore\ x = 10, y = 4$

(b) (i) Total number of data = 50

Middle position = $\frac{50 + 1}{2}$

= 25.5^{th} position

$\therefore$ Median = mean of the data in the 25^{th} and the 26^{th} position

= $\frac{2 + 2}{2}$

= 2

(ii) Mode = 2

(c) Number of years with at most p major hurricanes = 36% × 50

= $\frac{36}{100} \times 50$

= 18

Since 5 + 13 = 18,

$\therefore p = 1$

14. (a)

$$\text{Mean} = 2.2$$

$$\frac{4 \times 0 + 6 \times 1 + 3 \times 2 + x \times 3 + 3 \times 4 + 2 \times 5}{4 + 6 + 3 + x + 3 + 2} = 2.2$$

$$\frac{6 + 6 + 3x + 12 + 10}{x + 18} = 2.2$$

$$3x + 34 = 2.2(x + 18)$$

$$3x + 34 = 2.2x + 39.6$$

$$0.8x = 5.6$$

$$x = 7$$

(b) Total number of data = 4 + 6 + 3 + 7 + 3 + 2

= 25

Middle position = $\frac{25 + 1}{2}$

= 13^{th} position

The median is the data in the 13^{th} position.

The greatest possible value of x occurs when

$4 + 6 + (3 - 1) = x + 3 + 2$

$12 = x + 5$

$\therefore x = 7$

(c) Since modal number = 3,

$\therefore$ Smallest possible value of $x = 7$

15. (i) Total number of students

$= x + 1 + x - 2 + x + 2 + x + x - 2 + x - 4 + x - 3$

$= 7x - 8$

$$\text{Mean} = \frac{(x + 1) \times 0 + (x - 2) \times 1 + (x + 2) \times 2 + x \times 3 + (x - 2) \times 4 + (x - 4) \times 5 + (x - 3) \times 6}{7x - 8}$$

$$= \frac{x - 2 + 2x + 4 + 3x + 4x - 8 + 5x - 20 + 6x - 18}{7x - 8}$$

$$= \frac{21x - 44}{7x - 8}$$

Mode = 2

$$\therefore \frac{21x - 44}{7x - 8} = 2$$

$$21x - 44 = 2(7x - 8)$$

$$21x - 44 = 14x - 16$$

$$7x = 28$$

$$x = 4$$

(ii)

Number of books	0	1	2	3	4	5	6
Number of students	5	2	6	4	2	0	1

Total number of data = 7(4) – 8

= 20

Middle position = $\frac{20 + 1}{2}$

= 10.5^{th} position

$\therefore$ Median = mean of the data in the 10^{th} and the 11^{th} position

= $\frac{2 + 2}{2}$

= 2

Review Exercise 14

1. (a) Mean = $\frac{8 + 11 + 14 + 13 + 14 + 9 + 15}{7}$

= $\frac{84}{7}$

= 12

Total number of data = 7

Middle position = $\frac{7 + 1}{2}$

= 4^{th} position

Rearrange the numbers in ascending order,

8, 9, 11, 13, 14, 14, 15

$\therefore$ Median = data in the 4^{th} position

= 13

Mode = 14

(b) Mean = $\frac{88 + 93 + 85 + 98 + 102 + 98}{6}$

= $\frac{564}{6}$

= 94

Total number of data = 6

Middle position = $\frac{6 + 1}{2}$

= 3.5^{th} position

Rearranging the numbers in ascending order,

85, 88, 93, 98, 98, 102

$\therefore$ Median = mean of data in the 3^{rd} position and 4^{th} position

= $\frac{93 + 98}{2}$

= 95.5

Mode = 98

2. Since mean of 16 numbers = $\dfrac{\text{sum of the 16 numbers}}{16}$

then sum of the 16 numbers $= 16 \times \text{mean}$
$= 16 \times 7$
$= 112$

Since mean of 6 numbers = $\dfrac{\text{sum of the 6 numbers}}{6}$,

then sum of the 6 numbers $= 6 \times \text{mean}$
$= 6 \times 2$
$= 12$

Sum of the other set of 10 numbers $= 112 - 12$
$= 100$

Mean of the other set of 10 numbers, $x = \dfrac{100}{10}$
$= 10$

3. Since mode = 29 and $a < b$,
a or $b = 29$, as 29 will be the value that occurs most frequently.
Total number of data = 10

Middle position $= \dfrac{10 + 1}{2}$
$= 5.5^{\text{th}}$ position

Consider $a = 29$.
Rearranging the numbers in ascending order,
22, 24, 24, 25, 28, 29, 29, 29, 34, b
$\therefore$ Median = mean of the data in the 5^{th} position and 6^{th} position
$= \dfrac{28 + 29}{2}$
$= 28.5 \neq 27$

Hence, $b = 29$.
Rearranging the numbers in ascending order,
a, 22, 24, 24, 25, 28, 29, 29, 29, 34

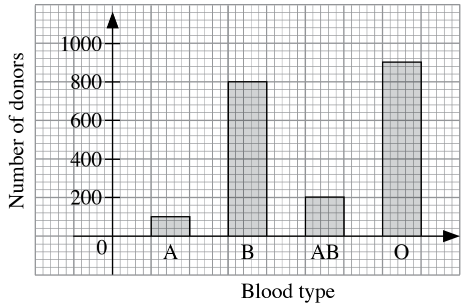
By observation, $a = 26$, since median $= \dfrac{26 + 28}{2} = 27$

$\therefore a = 26, b = 29$

4. **(a)** **(i)** $h + 800 + 200 + k = 2000$
$h + k + 1000 = 2000$
$h + k = 1000$

(ii) $h + k = 1000$ — (1)
$9h = k$ — (2)
Substitute (2) into (1):
$h + 9h = 1000$
$10h = 1000$
$h = 100$
Substitute $h = 1000$ into (2):
$9(100) = k$
$k = 900$
$\therefore h = 100, k = 900$

(b) **(i)**



(ii) Mode = O

5. **(i)** Mean annual increment

$= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{2 \times 3 + 1 \times 4 + 6 \times 5 + 1 \times 6 + 2 \times 7 + 7 \times 8 + 1 \times 15}{20}$

$= \dfrac{131}{20}$

$= 6.55\%$

(ii) Total number of data = 20

Middle position $= \dfrac{20 + 1}{2}$
$= 10.5^{\text{th}}$ position

$\therefore$ Median annual increment
= mean of the data in the 10^{th} and 11^{th} position
$= \dfrac{6 + 7}{2}$
$= 6.5\%$

(iii) Modal annual increment = 8%

6. **(a)** **(i)** Mean current

$$= \frac{\begin{array}{l}52 + 68 + 69 + 69 + 70 + 70 + 70 + 71 + 72 \\ + 72 + 72 + 72 + 73 + 74 + 74 + 76 + 76 + 77 \\ + 77 + 77 + 84 + 84 + 85 + 90 + 93\end{array}}{25}$$

$= \dfrac{1867}{25}$

$= 74.68$ A

(ii) Total number of data = 25

Middle position $= \dfrac{25 + 1}{2}$
$= 13^{\text{th}}$ position

$\therefore$ Median = data in the 13^{th} position
$= 73$ A

(iii) Mode = 72 A

(b) The current flowing through 25 electrical conductors ranges from 52 A to 93 A. The current is clustered around 68 A to 77 A. There is an extreme value of 52 A.

7. **(a)** **(i)** $21 + x + y + 18 + 17 = 100$

$$x + y + 56 = 100$$
$$x + y = 44 \text{ (shown)}$$

(ii) Mean of the distribution

$$= \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{21 \times 1 + x \times 2 + y \times 3 + 18 \times 4 + 17 \times 5}{100}$$
$$= \frac{21 + 2x + 3y + 72 + 85}{100}$$
$$= \frac{2x + 3y + 178}{100}$$
$$\frac{2x + 3y + 178}{100} = 2.9$$
$$2x + 3y + 178 = 290$$
$$2x + 3y = 112 \text{ (shown)}$$

(iii) $x + y = 44$ — (1)

$2x + 3y = 112$ — (2)

$3 \times (1)$: $3x + 3y = 132$ — (3)

$(3) - (2)$:

$(3x + 3y) - (2x + 3y) = 132 - 112$

$3x + 3y - 2x - 3y = 20$

$x = 20$

Substitute $x = 20$ into (1):

$20 + y = 44$

$y = 24$

$\therefore x = 20, y = 24$

(b) **(i)** Total number of data = 100

Middle position $= \dfrac{100 + 1}{2}$

$= 50.5^{\text{th}}$ position

$\therefore$ Median

= mean of the data in the 50^{th} position and 51^{th} position

$$= \frac{3 + 3}{2}$$
$$= 3$$

(ii) Mode = 3

8. **(a)** Mean number of songs

$$= \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{5 \times 10 + 12 \times 15 + 4 \times 20 + m \times 25 + 5 \times 30}{5 + 12 + 4 + m + 5}$$
$$= \frac{25m + 460}{m + 26}$$
$$\frac{25m + 460}{m + 26} = 20.25$$
$$25m + 460 = 20.25(m + 26)$$
$$25m + 460 = 20.25m + 526.5$$
$$4.75m = 66.5$$
$$m = 14$$

(b) We write the data as follows:

$$\underbrace{10, ..., 10}_{5}, \underbrace{15, ..., 15}_{12}, \underbrace{20, ..., 20}_{4}, \underbrace{25, ..., 25}_{m}, \underbrace{30, ..., 30}_{5}$$

Since median = 20,

the smallest possible value of m occurs when the median is here.

$5 + 12 = (4 - 1) + m + 5$

$17 = m + 8$

$m = 9$

(c) Since mode = 15,

$\therefore$ greatest possible value of $m = 11$

9. **(a)** **(i)** Mean score of team Cheetah $= \dfrac{65 + 95 + 32 + 96 + 88}{5}$

$$= \frac{376}{5}$$
$$= 75.2$$

Mean score of team Jaguar $= \dfrac{50 + 90 + 65 + 87 + 87}{5}$

$$= \frac{379}{5}$$
$$= 75.8$$

Mean score of team Puma $= \dfrac{90 + 85 + 46 + 44 + 80}{5}$

$$= \frac{345}{5}$$
$$= 69$$

I would join team Jaguar as the team has the highest mean score.

(ii) Total number of data = 5

Middle position $= \dfrac{5 + 1}{2}$

$= 3^{\text{rd}}$ position

Rearranging the scores of team Cheetah,

32, 65, 88, 95, 96

Median score of team Cheetah = data in the 3^{rd} position

= 88

Rearranging the scores of team Jaguar,

50, 65, 87, 87, 90

Median score of team Jaguar = data in the 3^{rd} position

= 87

Rearranging the scores of team Puma,

44, 46, 80, 85, 90

Median score of team Puma = data in the 3^{rd} position

= 80

I would join team Cheetah as the team has the highest median score.

(b) I would report the median score as the median score of 87 is higher than the mean score of 75.8.

Challenge Yourself

1. Let the 4 numbers be a, b, c and d, where $a \leqslant b \leqslant c \leqslant d$.

Since mean $= \dfrac{\text{sum of the 4 numbers}}{4}$,

then sum of the 4 numbers $= 4 \times \text{mean}$

$$= 4 \times (x + y + 5)$$
$$= 4x + 4y + 20$$

$\therefore a + b + c + d = 4x + 4y + 20$ — (1)

Total number of data = 4

Middle position $= \dfrac{4 + 1}{2}$

$= 2.5^{\text{th}}$ position

Arranging the numbers in ascending order,

a, b, c, d

Median = mean of the data in the 2^{nd} position and 3^{rd} position

$$= \frac{b + c}{2}$$

$\therefore \dfrac{b + c}{2} = x + y$ — (2)

Since mode $= x$,

x occurs twice. x cannot occur 3 times because median

$= \dfrac{x + x}{2} = x \neq x + y.$

Since the 4 numbers are whole numbers,

$\therefore a = b = x$

Substitute $b = x$ into (2):

$$\frac{x + c}{2} = x + y$$
$$x + c = 2x + 2y$$
$$c = x + 2y$$

Substitute $a = b = x$ and $c = x + 2y$ into (1):

$$x + x + x + 2y + d = 4x + 4y + 20$$
$$d + 3x + 2y = 4x + 4y + 20$$
$$d = x + 2y + 20$$

$\therefore$ The numbers are $x, x, x + 2y, x + 2y + 20$

2. $a + \dfrac{1}{2}b = 13 - \dfrac{1}{2}e$ — (1)

$c + \dfrac{1}{2}e + f = 8 - \dfrac{1}{2}b - d$ — (2)

(1) + (2):

$$a + \frac{1}{2}b \quad + \quad c + \frac{1}{2}e + f \quad = \quad 13 - \frac{1}{2}e \quad + \quad 8 - \frac{1}{2}b - d$$

$$a + \frac{1}{2}b + c + \frac{1}{2}e + f = 21 - \quad b - d - \frac{1}{2}e$$

$$a + b + c + d + e + f = 21 \quad \frac{1}{2}$$

Mean of a, b, c, d, e and $f = \dfrac{a + b + c + d + e + f}{6}$

$$= \frac{21}{6}$$
$$= 3.5$$

3. Given: There are 3 (and only 3) boys with a height of 183 cm and one (and only one) boy with a height of 187 cm.

Since mean $= \dfrac{\text{sum of the heights of the 9 boys}}{9}$,

then sum of the heights of the 9 boys $= 9 \times \text{mean}$

$$= 9 \times 183$$
$$= 1647 \text{ cm}$$

Since 3 boys have a height of 183 cm, and the tallest boy has a height of 187 cm,

Sum of the heights of the remaining boys $= 1647 - (3 \times 183) - 187$

$$= 1647 - 549 - 187$$
$$= 911 \text{ cm}$$

Total number of data = 9

Middle position $= \dfrac{9 + 1}{2}$

$= 5^{\text{th}}$ position

Median = 183 cm and this data is in the 5^{th} position.

Since mode = 183 cm, it occurs 3 times. And the other heights can occur at most 2 times.

We let the heights of 4 boys be 186 cm, 186 cm, 185 cm and 182 cm.

Least possible height of the shortest boy

$= 911 - 186 - 186 - 185 - 182$

$= 172$ cm

Check:

Rearranging the heights of the 9 boys in ascending order:

172, 182, 183, 183, 183, 185, 186, 186, 187

Mean $= \dfrac{172 + 182 + 183 + 183 + 183 + 185 + 186 + 186 + 187}{9}$

$$= \frac{1647}{9}$$
$$= 183 \text{ cm}$$

Median = data in the 5^{th} position

$= 183$ cm

Mode = 183 cm

Chapter 15 Probability

TEACHING NOTES

Suggested Approach

Teachers may begin the lesson by first arousing the students' interest in this topic. This new topic can be introduced by using the chapter opener on Page 264 and discussing statements that are often used in our daily life (see Thinking Time on page 265).

Section 15.1: Introduction to Probability

From the statements discussed in our daily life (see Thinking Time on page 265), teachers can build upon them and guide the students to determine the chance of each event happening. This measure of chance is the definition of probability.

This can then lead to relating the chance of any event happening to a number line taking on values between 0 and 1 inclusive. Teachers should further discuss the reason for a 'certain' event to take the value of 1 as well as the reason for an 'impossible' event to take the value of 0.

At the end of this section, a simple class activity can be carried out by encouraging students to form statements to describe an event. Words such as 'unlikely', 'likely', 'impossible' or 'certain' are encouraged to be used in the statements. Students can then mark the chance of the event occurring on a number line. At the end of this section, students should be able to define probability as a measure of chance.

Section 15.2: Sample Space

Teachers can do a simple experiment such as tossing a coin to introduce the words 'outcomes' and 'sample space'. Going through the different experiments listed in the table on Page 266 can help to reinforce the meanings of 'outcomes' and 'sample space'.

For the last experiment, teachers are to let the students know that there is a difference between drawing the first and second black ball. Thus, there is a need to differentiate the two black balls. Similarly, there is a need to differentiate the three white balls. Hence, the outcomes are two individual black balls and three individual white balls.

Teachers can assess students' understanding of the terms using the Practise Now exercises available.

Section 15.3: Probability of Single Events

To start off this section, students should attempt the two activities (see Investigation: Tossing a Coin and Investigation: Rolling a Die). The terms 'experimental probability' and 'theoretical probability' and their relationship should be highlighted and explained to students. Students are expected to be able to conclude that as the number of trials increase, the experimental probability of an outcome occurring tends towards the theoretical probability of the outcome happening.

At the same time, teachers can illustrate the meanings of 'fair' and 'unbiased'. If any of the two terms are used, then the chance of any outcome happening in an event is exactly equal.

In the previous sections, students would have learnt events with a probability of 0 or 1. Here, they are to grasp that for any event E, $0 \leqslant P(E) \leqslant 1$ and later $P(\text{not } E) = 1 - P(E)$.

As a recommended technique for solving problems involving probability, teachers should encourage the students to always list all the possible outcomes in a simple chance situation to calculate the probability. Doing so will allow better visualisation of the outcomes for a particular event to happen.

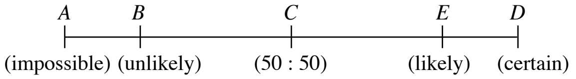
Challenge Yourself

For Question 1, teachers can provide a hint to students to focus solely on the picture cards by recalling the number of it in a standard pack.

WORKED SOLUTIONS

Thinking Time (Page 265)

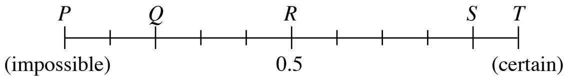
1.



2. It takes a value of 0.5.

3. Suggested answers:

P: The sun will rise from the west at least once a year.
Q: There is a 20 percent chance that it will rain tomorrow.
R: A lady will give birth to a boy if she is pregnant with a child.
S: A red ball is drawn from a bag which contains 9 red balls and 1 black ball.
T: A die will land on a number between 1 to 6 inclusive.



Investigation (Tossing a Coin)

1. No.

2. **(i)** *Teachers may guide the students to fill in the necessary information in the table.*

(ii) The fraction should be close to $\frac{1}{2}$.

(iii) The results each classmate got are likely to be different. The results obtained are those of a random experiment.

3. **(a)** *Teachers may guide the students to fill in the necessary information in the table.*

(b) *Teachers may guide the students to fill in the necessary information in the table.*

4. Yes, the probabilities are approaching $\frac{1}{2}$ when there are more tosses.

5. No, we can only expect to get close to it as every outcome cannot be predicted with certainty before a coin is tossed.

Investigation (Rolling a Dice)

1. *Teacher may ask students to visit the website:* *http://www.shinglee.com.sg/StudentResources/and open the spreadsheet 'Rolling a Die' to obtain the outcomes so as to fill in the necessary information for the table.*

2. *Teacher may guide the students to fill in the necessary information for the table.*

3. Yes, the probabilities will approach $\frac{1}{6}$ when there are more rolls.

4. No, we can only expect to get close to it as every outcome cannot be predicted with certainty before a die is rolled.

Thinking Time (Page 273)

1. The probability of D occurring is 1. Event D will definitely occur.

2. The probability of A occurring is 0. Event A will never occur.

3. No.

Performance Task (Page 273)

Suggested answers of other real-life applications of probability theory are in

- risk assessment,
- trades on financial markets,
- reliability of consumer products etc.

Teachers may wish to group the students for this task. Each group can be responsible to do research on one of the above applications and then present their findings to the class.

Practise Now 1

1. The spinner has the colours Red, Orange, Purple, Yellow and Green. i.e. the sample space consists of the colours Red, Orange, Purple, Yellow and Green.
Total number of possible outcomes = 5

Practise Now 2

(a) Let B_1, B_2, B_3, B_4, B_5 represent the 5 blue marbles;
R_1, R_2, R_3, R_4 represent the 4 red marbles.
The sample space consists of B_1, B_2, B_3, B_4, B_5, R_1, R_2, R_3 and R_4.
Total number of possible outcomes = 9

(b) The sample space consists of N_1, A_1, T, I, O, N_2, A_2 and L.
Total number of possible outcomes = 8

(c) The sample space consists of the serial numbers 357, 358, 359, ..., 389.
Total number of possible outcomes
= first 389 receipts – first 356 receipts
= 389 – 356
= 33

Practise Now 3

Total number of possible outcomes = 24 – 9
= 15

(i) P(drawing a '21') = $\frac{1}{15}$

(ii) There are 7 odd numbers from 10 to 24, i.e. 11, 13, 15, 17, 19, 21 and 23.

P(drawing an odd number) = $\frac{7}{15}$

(iii) There are 10 composite numbers from 10 to 24, i.e. 10, 12, 14, 15, 16, 18, 20, 21, 22 and 24.

P(drawing a composite number) $= \frac{10}{15}$
$= \frac{2}{3}$

(iv) There are no perfect cubes from 10 to 24.

P(drawing a perfect cube) = $\frac{0}{15}$
= 0

Practise Now 4

Total number of possible outcomes = 52

(i) There are 26 red cards in the pack.

$$\text{P(drawing a red card)} = \frac{26}{52} = \frac{1}{2}$$

(ii) There are 4 aces in the pack, i.e. the ace of clubs, the ace of diamonds, the ace of hearts and the ace of spade.

$$\text{P(drawing an ace)} = \frac{4}{52} = \frac{1}{13}$$

(iii) There is only one three of clubs in the pack.

$$\text{P(drawing the three of clubs)} = \frac{1}{52}$$

(iv) Since there is only one three of clubs in the pack, there are $52 - 1 = 51$ cards which are not the three of clubs.

$$\text{P(drawing a card which is not the three of clubs)} = \frac{51}{52}$$

Practise Now 5

1. Total number of letters = 8

(i) There is 1 'D'.

$$\text{P(a 'D' is chosen)} = \frac{1}{8}$$

(ii) There are 6 consonants, i.e. 1 'C', 1 'H', 1 'L', 1 'D', 1 'R', and 1 'N',

$$\text{P(letter chosen is a consonant)} = \frac{6}{8} = \frac{3}{4}$$

(iii) Method 1:

There are 2 vowels, i.e. 1 'I' and 1 'E'.

$$\text{P(letter chosen is not a consonant)} = \frac{2}{8} = \frac{1}{4}$$

Method 2:

P(letter chosen is a consonant)

= 1 – P(letter chosen is a consonant)

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

2. Total number of possible outcomes $= 9 + 6 + 4 + 5 = 24$

(i) There are 4 purple marbles.

$$\text{P(drawing a purple marble)} = \frac{4}{24} = \frac{1}{6}$$

(ii) There are $9 + 5 = 14$ red or blue marbles.

$$\text{P(drawing a red or a blue marble)} = \frac{14}{24} = \frac{7}{12}$$

(iii) There are no white marbles.

P(drawing a white marble) = 0

(iv) Method 1:

There are 24 marbles that are not white.

$$\text{P(drawing a marble that is not white)} = \frac{24}{24} = 1$$

Method 2:

P(drawing a marble that is not white)

= 1 – P(drawing a marble that is white)

= 1 – 0

= 1

3. Method 1:

$$\text{P(drawing a red ball)} = \frac{1}{3}$$

$$\frac{\text{Number of red balls}}{\text{Total number of balls}} = \frac{1}{3}$$

$$\therefore \text{Number of red balls} = \frac{1}{3} \times 24 = 8$$

$$\text{P(drawing a green ball)} = \frac{1}{6}$$

$$\frac{\text{Number of green balls}}{\text{Total number of balls}} = \frac{1}{6}$$

$$\therefore \text{Number of green balls} = \frac{1}{6} \times 24 = 4$$

$$\therefore \text{Number of blue balls} = 24 - 8 - 4 = 12$$

Method 2:

P(drawing a blue ball)

= 1 – P(drawing a red ball) – P(drawing a green ball)

$= 1 - \frac{1}{3} - \frac{1}{6}$

$= \frac{1}{2}$

$$\frac{\text{Number of blue balls}}{\text{Total number of balls}} = \frac{1}{2}$$

$$\therefore \text{Number of blue balls} = \frac{1}{2} \times 24 = 12$$

Exercise 15A

1. The dart board has the numbers 1, 2, 3, 4, 5 and 6.
i.e. the sample space consists of the numbers 1, 2, 3, 4, 5 and 6.
Total number of possible outcomes = 6

2. (a) The die has the labels 2, 3, 4 and 5.
i.e. the sample space consists of the labels 2, 3, 4, 5 and 6.
Total number of possible outcomes = 5

(b) The box has the cards A, B, C, D, E, F, G, H, I and J.

i.e. the sample space consists of cards A, B, C, D, E, F, G, H, I and J.

Total number of possible outcomes = 10

(c) Let R_1, R_2, R_3, R_4, R_5 represents the 5 red discs;

B_1, B_2, B_3 represent the 3 blue discs;

G_1, G_2 represent the 2 green discs.

The sample space consists of R_1, R_2, R_3, R_4, R_5, B_1, B_2, B_3, G_1 and G_2.

Total number of possible outcomes = 10

(d) The sample space consists of T, E_1, A, C, H, E_2 and R.

Total number of possible outcomes = 7

(e) The sample space consists of the numbers 100, 101, 102, ..., 999.

Total number of possible outcomes

= first 999 numbers – first 99 numbers

= 999 – 99

= 900

3. Total number of possible outcomes = 8

(i) There are 3 '7's.

P(getting a '7') = $\frac{3}{8}$

(ii) There are 2 '3's and 1 '4'.

P(getting a '3' or '4') = $\frac{3}{8}$

(iii) There are 8 numbers less than 10, i.e. 2, 3, 3, 4, 7, 7, 7 and 9.

P(getting a number less than 10) = $\frac{8}{8}$

= 1

(iv) Method 1:

There are 7 numbers that are not '2', i.e. 3, 3, 4, 7, 7, 7 and 9.

P(getting a number which is not '2') = $\frac{7}{8}$

Method 2:

There is 1 '2'.

P(getting a '2') = $\frac{1}{8}$

P(getting a number which is not '2')

= 1 – P(getting a '2')

= $1 - \frac{1}{8}$

= $\frac{7}{8}$

4. Total number of possible outcomes = 22 – 9

= 13

(i) There are 7 even numbers from 10 to 22, i.e. 10, 12, 14, 16, 18, 20 and 22.

P(drawing an even number) = $\frac{7}{13}$

(ii) There are 7 numbers between 13 and 19 inclusive, i.e. 13, 14, 15, 16, 17, 18 and 19.

P(drawing a number between 13 and 19 inclusive) = $\frac{7}{13}$

(iii) There are 3 prime numbers less than 18, i.e. 11, 13 and 17.

P(drawing a prime number that is less than 18) = $\frac{3}{13}$

(iv) There are no numbers greater than 22.

P(drawing a number greater than 22) = 0

(v) There are 3 numbers divisible by 4, i.e. 12, 16 and 20.

P(drawing a number that is divisible by 4) = $\frac{3}{13}$

5. Total number of possible outcomes = 52

(i) There is only one ace of spades in the pack.

P(drawing the ace of spades) = $\frac{1}{52}$

(ii) P(drawing a heart or a club) = $\frac{26}{52}$

= $\frac{1}{2}$

(iii) There are 3 × 4 = 12 picture cards in the pack.

P(drawing a picture card) = $\frac{12}{52}$

= $\frac{3}{13}$

(iv) Method 1:

There are 10 × 4 = 40 non-picture cards in the pack.

P(drawing a non-picture card) = $\frac{40}{52}$

= $\frac{10}{13}$

Method 2:

P(getting a non-picture card)

= 1 – P(getting a picture card)

= $1 - \frac{3}{13}$

= $\frac{10}{13}$

6. Total number of possible outcomes = 11

(i) There is 1 'A' in the cards.

P(card shows the letter 'A') = $\frac{1}{11}$

(ii) There are 2 'B's in the cards.

P(card shows the letter 'B') = $\frac{2}{11}$

(iii) There are 4 vowels in the cards, i.e. 1 'O', 1 'A' and 2 'I's.

P(card shows a vowel) = $\frac{4}{11}$

(iv) Method 1:

There are 7 consonants, i.e. 1 'P', 1 'R', 2 'B's, 1 'L', 1 'T' and 1 'Y'.

P(cards shows a consonant) = $\frac{7}{11}$

Method 2:

P(card shows a consonant)

= 1 – P(card shows a vowel)

= $1 - \frac{4}{11}$

= $\frac{7}{11}$

7. Total number of possible outcomes = 5

(i) There is one sector with the label ♥.

P(stops at a sector whose label is ♥) = $\frac{1}{5}$

(ii) There are 3 sectors with a letter of the English alphabet, i.e. A, V and F.

P(stops at a sector whose label is a letter of the English alphabet)

= $\frac{3}{5}$

(iii) There is one sector with a vowel, i.e. A.

P(stops at a sector whose label is a vowel) = $\frac{1}{5}$

(iv) There are 2 sectors with a consonant, i.e. V and F.

P(stops at a sector whose label is a consonant) = $\frac{2}{5}$

8. Total number of possible outcomes = 4

(i) There is one caramel in the bag.

P(candy is a caramel) = $\frac{1}{4}$

(ii) There are 2 pieces that are either a chocolate or gummy in the bag.

P(candy is either a chocolate or a gummy) = $\frac{2}{4}$

= $\frac{1}{2}$

(iii) **Method 1:**

There are 3 pieces that are not a licorice, i.e. caramel, chocolate and gummies.

P(candy is not a licorice) = $\frac{3}{4}$

Method 2:

There is one licorice.

P(candy is a licorice) = $\frac{1}{4}$

P(candy is not licorice)

= 1 – P(candy is a licorice)

= $1 - \frac{1}{4}$

= $\frac{3}{4}$

9. Total number of possible outcomes = 40

There are 15 vouchers with a value of \$100 each.

P(voucher has a value of \$100) = $\frac{15}{40}$

= $\frac{3}{8}$

10. (i) Total number of possible outcomes = 30

There are 9 + 12 = 21 males in the group.

P(person is a male) = $\frac{21}{30}$

= $\frac{7}{10}$

(ii) **Method 1:**

There are 6 + 12 + 3 = 21 people in the group that is either a woman, a boy or a girl.

P(person is either a woman, a boy or a girl) = $\frac{21}{30}$

= $\frac{7}{10}$

Method 2:

There are 9 men in the group.

P(person is a man) = $\frac{9}{30}$

= $\frac{3}{10}$

P(person is either a woman, a boy or a girl)

= P(person is not a man)

= 1 – P(person is a man)

= $1 - \frac{3}{10}$

= $\frac{7}{10}$

11. The sample space consists of the numbers, 10, 11, 12, ..., 99.

Total number of possible outcomes = 99 – 9

= 90

(i) There are 10 numbers less than 20, i.e. 10, 11, 12, ..., 19.

P(number is less than 20) = $\frac{10}{90}$

= $\frac{1}{9}$

(ii) There are 6 perfect squares that are two-digit numbers, i.e. 16, 25, 36, 49, 64, 81.

P(number is a perfect square) = $\frac{6}{90}$

= $\frac{1}{15}$

12. Total number of possible outcomes = 54

(i) There are 26 red cards in the pack.

P(drawing a red card) = $\frac{26}{54}$

= $\frac{13}{27}$

(ii) There are 4 '2' cards in the pack.

P(drawing a two) = $\frac{4}{54}$

= $\frac{2}{27}$

(iii) There are 2 joker cards in the pack.

P(drawing a joker) = $\frac{2}{54}$

= $\frac{1}{27}$

(iv) There are 4 + 4 = 8 cards that are either a queen or a king in the pack.

P(drawing a queen or a king) = $\frac{8}{54}$

= $\frac{4}{27}$

13. Total number of possible outcomes $= 52 - 13$
$= 39$

(i) There are 13 black cards in the pack, i.e. 13 spades.

$$\text{P(drawing black card)} = \frac{13}{39} = \frac{1}{3}$$

(ii) There are 13 diamonds in the pack.

$$\text{P(drawing a diamond)} = \frac{13}{39} = \frac{1}{3}$$

(iii) There are $3 \times 3 = 9$ picture cards in the pack.

$$\text{P(drawing a picture card)} = \frac{9}{39} = \frac{3}{13}$$

(iv) There are 3 aces in the pack, i.e. the ace of hearts, the ace of diamonds and the ace of spades.

$$\text{P(drawing an ace)} = \frac{3}{39} = \frac{1}{13}$$

$$\text{P(drawing a card which is not an ace)} = 1 - \text{P(drawing an ace)} = 1 - \frac{1}{13} = \frac{12}{13}$$

14. **(i)** The sample space in column *A* consists of the integers 0, 1, 2, 3, 4 and 5.

Total number of possible outcomes = 6

$$\text{P(number in colum } A \text{ is a 4)} = \frac{1}{6}$$

(ii) The sample space in column *B* consists of the integers 0, 1, 2, ..., 9.

Total number of possible outcomes = 10

$$\text{P(number in column } B \text{ is an 8)} = \frac{1}{10}$$

(iii) There are 6 numbers that are less than 6 in column *A*.

$$\text{P(number in column } A \text{ is less than 6)} = \frac{6}{6} = 1$$

(iv) There are 4 numbers that are greater than 5 in column *B*, i.e. 6, 7, 8 and 9.

$$\text{P(number in column } B \text{ is greater than 5)} = \frac{4}{10} = \frac{2}{5}$$

15. Total number of possible outcomes $= 2 \times 12$
$= 24$

$$\text{P(draw a pair of tinted lenses)} = \frac{8}{24} = \frac{1}{3}$$

P(draw a pair of non-tinted lenses)
= 1 – P(draw a pair of tinted lenses)

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

16. **(a)** Total number of possible outcomes $= 26 + 62 + 8 + 9 + 12$
$= 117$

(i) There are 62 teachers in the school.

$$\text{P(school personnel is a teacher)} = \frac{62}{117}$$

(ii) There are 26 management staff in the school.

$$\text{P(school personnel is a management staff)} = \frac{26}{117} = \frac{2}{9}$$

(iii) There are $9 + 12 = 21$ staff that are either an administrative or maintenance staff in the school.

P(school personnel is an administrative or maintenance staff)

$$= \frac{21}{117} = \frac{7}{39}$$

(b) Total number of possible outcomes $= 117 - 2 - 1$
$= 114$

(i) There are $9 - 1 = 8$ administrative staff in the school.

$$\text{P(school personnel is an administrative staff)} = \frac{8}{114} = \frac{4}{57}$$

(ii) There are 8 laboratory staff in the school.

$$\text{P(school personnel is a laboratory staff)} = \frac{8}{114} = \frac{4}{57}$$

P(school personnel is not a laboratory staff)
= 1 – P(school personnel is a laboratory staff)

$$= 1 - \frac{4}{57} = \frac{53}{57}$$

17. Total number of possible outcomes = 117

(i) P(pair of socks is yellow) = $\frac{2}{9}$

$\frac{\text{Number of yellow pairs of socks}}{\text{Total number of pairs of socks}} = \frac{2}{9}$

$\therefore$ Number of yellow pairs of socks $= \frac{2}{9} \times 117$

$= 26$

(ii) P(pair of socks is neither yellow nor grey)

= 1 – P(pair of socks is yellow) – P(pair of socks is grey)

$= 1 - \frac{2}{9} - \frac{3}{13}$

$= \frac{64}{117}$

$\frac{\text{Number of pairs of socks neither yellow nor grey}}{\text{Total number of pairs of socks}} = \frac{64}{117}$

$\therefore$ Number of pairs of sock neither yellow nor grey

$= \frac{64}{117} \times 117$

$= 64$

18. The sample space consists of the question numbers 1, 2, 3, ..., 80.

Total number of questions = 80

(i) There are 9 question numbers containing only a single digit, i.e. 1, 2, 3, ..., 9.

P(question number contains only a single digit) = $\frac{9}{80}$

(ii) There are 13 question numbers greater than 67, i.e. 68,69, 70, ..., 80.

P(question number is greater than 67) = $\frac{13}{80}$

(iii) There are 7 + 9 = 16 question numbers containing exactly one '7',

i.e. 7, 17, 27, 37, 47, 57, 67, 70, 71, 72, 73, 74, 75, 76, 78 and 79.

P(question number contains exactly one '7') = $\frac{16}{80}$

$= \frac{1}{5}$

(iv) There are 8 question numbers divisible by both 2 and 5, i.e. 10, 20, 30, 40, 50, 60, 70 and 80.

P(question number is divisible by both 2 and 5) = $\frac{8}{80}$

$= \frac{1}{10}$

19. The sample space consists of the two-digit numbers

23, 25, 27, 32, 35, 37, 52, 53, 57, 72, 73 and 75.

Total number of possible outcomes = 12

(i) There are 3 numbers divisible by 4, i.e. 32, 52 and 72.

P(two-digit number is divisible by 4) = $\frac{3}{12}$

$= \frac{1}{4}$

(ii) There are 4 prime numbers, i.e. 23, 37, 53 and 73.

P(two-digit number is a prime number) = $\frac{4}{12}$

$= \frac{1}{3}$

20. Let the probability of getting a '1' be x.

P(getting a '3') = $2x$

P(getting a '2') = $3(2x)$

$= 6x$

P(getting a '4') = $6x$

$\therefore x + 2x + 6x + 6x = 1$

$15x = 1$

$\therefore x = \frac{1}{15}$

2 and 3 are prime numbers.

P(getting a prime number) = $6x + 2x$

$= 8x$

$= 8 \left(\frac{1}{15}\right)$

$= \frac{8}{15}$

Review Exercise 15

1. (a) (i) The sample space consists of the numbers 5, 6, 8, 56, 58, 65, 68, 85, 86, 568, 586, 658, 685, 856, 865.

(ii) Total number of possible outcomes = 15

(b) (i) There are 6 two-digit numbers, i.e. 56, 58, 65, 68, 85 and 86.

P(number consists of two digits) = $\frac{6}{15}$

$= \frac{2}{5}$

(ii) There are 5 numbers that are a multiple of 5, i.e. 5, 65, 85, 685 and 865.

P(number is a multiple of 5) = $\frac{5}{15}$

$= \frac{1}{3}$

2. The sample space consists of 1, 2, 3, 4, 5 and 6.

Total number of possible outcomes = 6

(i) There are 3 even numbers, i.e. 2, 4 and 6.

P(rolls an even number) = $\frac{3}{6}$

$= \frac{1}{2}$

(ii) There are 2 composite numbers, i.e. 4 and 6.

P(rolls a composite number) = $\frac{2}{6}$

$= \frac{1}{3}$

(iii) There is only one number divisible by 4, i.e. 4.

P(rolls a number divisible by 4) = $\frac{1}{6}$

3. Total number of possible outcomes = 26

(i) There is only one queen of hearts in the pack.

P(drawing the queen of hearts) = $\frac{1}{26}$

(ii) There is no jack of clubs in the pack.

P(drawing the jack or clubs) = $\frac{0}{26}$

$= 0$

(iii) There are 2 cards that are either the six of hearts or the seven of diamonds.

P(drawing either the six of hearts or the seven of diamonds)

$= \frac{2}{26}$

$= \frac{1}{13}$

(iv) There are 2 cards that are a nine in the pack, namely, the nine of hearts and the nine of diamonds.

P(drawing a card that is a nine) = $\frac{2}{26}$

$= \frac{1}{13}$

P(drawing a card that is not a nine)

= 1 – P(drawing a card that is a nine)

$= 1 - \frac{1}{13}$

$= \frac{12}{13}$

4. Total number of possible outcomes = 6

(i) There is only one sector that has an umbrella as a prize.

P(wins an umbrella) = $\frac{1}{6}$

(ii) There is only one sector that has voucher.

P(wins a voucher) = $\frac{1}{6}$

(iii) There are no sectors with a prize of \$100 cash.

P(wins \$100 cash) = $\frac{0}{6}$

$= 0$

Challenge Yourself

1. Let the number of cards in the smaller pile be x and number of cards in the bigger pile be $52 - x$.

Total number of picture cards in a standard pack of 52 playing cards = 12

$$\therefore \frac{4}{11}x + \frac{2}{15}(52 - x) = 12$$

$$60x + 22(52 - x) = 1980$$

$$60x + 1144 - 22x = 1980$$

$$38x = 836$$

$$\therefore x = 22$$

Number of cards in the bigger pile = 52 – 22

= 30

There are 22 cards in the smaller pile and 30 cards in the bigger pile.